Name:[redacted] Number:[redacted]
Instructor:... Section:[redacted]

Question 1.(30 points) Circle the correct answer:

(1) The domain of the function $f(x,y) = \ln(1 - \frac{x^2}{2} - \frac{y^2}{3})$ is

(a) open and bounded. ✓

(b) closed and bounded.

(c) open and unbounded.

(d) closed and unbounded.

$D = \{(x,y) \mid 1 - \frac{x^2}{2} - \frac{y^2}{3} > 0\}$

$D = \{(x,y) \in \mathbb{R}^2,\ 1 - \frac{x^2}{2} - \frac{y^2}{3} > 1$

$\frac{x^2}{2} + \frac{y^2}{3} < 1$

(2) Let $\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j}$ be the position vector of a moving particle, the acceleration is

$V(t) = -2\sin t\ \hat{i} + 2\cos t\ \hat{j}$

(a) $\mathbf{a} = 2\mathbf{T}$

(b) $\mathbf{a} = 2\mathbf{N}$

(c) $\mathbf{a} = \mathbf{T} + \mathbf{N}$

(d) $\mathbf{a} = \mathbf{T} + 2\mathbf{N}$

$a(t) = -2\cos t\ \hat{i} - 2\sin t\ \hat{j}$

$|V| = \sqrt{4\sin^2 + 4\cos^2} = \sqrt{4} = 2$

$T = -\sin + \cos$

$N = -\cos - \sin$

$T = \frac{V}{|V|}$

$N = \frac{dT/dt}{|dT/dt|}$

$V = 2\sin$

a

$V = -2\sin t + 2$

(3) Let $\nabla f(1,-1) = 2\mathbf{i} - \mathbf{j}$, the equation of the tangent line to the level curve $f(x,y) = 1$ at $(1,-1)$ is

$y - y_0 =$ $a f_x(x - x_0) + b f_y(y - y_0) = 0$

(a) $y = x - 1$

(b) $y = 2x - 1$.

(c) $y = 2x - 3$.

(d) $y = -2x + 3$.

???? (4) Assume that the equation $x^2 z + y\ln z = xy$ defines z as a function of x and y. At the point $(x,y,z) = (1,1,1)$, $z_x =$

$\frac{f_x}{f_z}$ $\frac{dz}{dx}$

(a) 1

(b) $\frac{1}{2}$

(c) $-\frac{1}{2}$

(d) -1

$\frac{f_z}{f_x}$

$\frac{\partial z}{\partial x} \rightarrow x^2 \frac{\partial z}{\partial x} + z2x + \frac{y}{z}\frac{\partial z}{\partial x} = y$

$\frac{\partial z}{\partial x}(x^2 + z2x + \frac{y}{z}) = y$

$(1 + 2 + 1) = 1$

$2x\frac{\partial z}{\partial x} + \frac{y}{z}\frac{\partial z}{\partial x} = y$

$\frac{\partial z}{\partial x}(2x + \frac{y}{z}) = y$

$2 + 1$

25.5

(5) The arc length parameter for the curve $\mathbf{r}(t) = (1 + 2t)\mathbf{i} + (2 - 2t)\mathbf{j} + t\mathbf{k}$ is

(a) $s(t) = 3t$

(b) $s(t) = 9t$

(c) $s(t) = t$

(d) $s(t) = 3t^2$

$L = \int |V(t)| \, dt$

$V(t) = 2\hat{i} - 2\hat{j} + 1$

$|V(t)| = \sqrt{4+4+1} = 3$

(6) Assume that $w = f(s^3 + t^2)$ and $f'(x) = e^x$ then $\frac{\partial w}{\partial s} =$

(a) $e^{s^3+t^2}$

(b) $3s^2 e^{s^3+t^2}$

(c) $s^3 e^{s^3+t^2}$

(d) $3s^2 e^{s^3}$

$f(s^3+t^2) = 3s^2 e$

(7) Let $f(x, y) = \sin(x^2 + y^2)$, then $f_{xx}(0, 0) =$

(a) 1

(b) 2

(c) -2

(d) 4

$f_x = 2x\cos(x^2+y^2)$

$f_{xx} = 2\cos(x^2+y^2) - 2x\sin(x^2+y^2) \cdot 2x$

$f_x = \cos(x^2+y^2)2x$

$f_{xx} = -2x\sin(x^2+y^2)2x + \cos(x^2+y^2)2$

(8) The function $f(x, y) = \begin{cases} \frac{x^2 y}{x^2+y^2} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$

(a) is continuous at $(0, 0)$

(b) is not continuous at $(0, 0)$

(c) $\lim_{(x,y)\to(0,0)}$ does not exist.

(d) None of the above.

$r^2 \cos^2\theta \, r\sin\theta$

(9) Let $\mathbf{r}(t) = t\mathbf{i} + \ln t\mathbf{j} + t^2\mathbf{k}$ be the position vector of a moving particle. The speed of the particle at $t = 1$ is

(a) 2

(b) 4

(c) 6

(d) $\sqrt{6}$

$\vec{V}(t) = 1\hat{i} + \frac{1}{t}\hat{j} + 2t\hat{k}$

$= 1\hat{i} + 1\hat{j} + 2\hat{k}$

$V(t) = i + \frac{1}{t} + 2t$

$S = |V(t)| = \sqrt{1 + \frac{1}{t^2} + 4t^2}$

$\sqrt{1+1+4}$

$S = |V|$

$V(t) = i + \frac{1}{t}j + 2t = 1 + 1 + 2$

$|V(t)| = \sqrt{1 + 1 + 2^2} = \sqrt{6}$

(10) Let $\mathbf{r}(t)$ be a curve in space such that $|\mathbf{r}(t)| = |\mathbf{r}'(t)| = 1$. Then

(a) $\mathbf{r}(t).\mathbf{v}(t) = 0$

(b) $\mathbf{v}(t).\mathbf{a}(t) = 0$

(c) $\mathbf{r}(t).\mathbf{r}(t) = \mathbf{v}(t).\mathbf{v}(t) = 1$

(d) All of the above.

(11) $\lim_{(x,y)\to(1,1)} \frac{xy-1}{x-1}$

(a) 1

(b) 2

(c) 0

(d) Does not exist.

$\frac{x^2-1}{x-1} = x+1 \Rightarrow 2$

$y=1$ $\frac{x-1}{x-1}$ 1

(12) Let $\mathbf{r}(t) = t\mathbf{i} + \cosh t\mathbf{j}$. The unit tangent vector $\mathbf{T}$ is

(a) $\mathbf{T} = \sinh t\mathbf{i} + \cosh t\mathbf{j}$

(b) $\mathbf{T} = \text{sech} t\mathbf{i} + \tanh t\mathbf{j}$

(c) $\mathbf{T} = \tanh t\mathbf{i} + \text{sech} t\mathbf{j}$

(d) None of the above

$V(t) = \hat{i} + \sinh t\,\hat{j}$

$\text{sech}\,t + \tanh t$

$\sqrt{1+\sinh^2 t} = \cosh t$

$\sinh^2 - \cosh^2 = 1$

(13) If $\mathbf{v}(t) = \mathbf{i} + \sqrt{2t}\mathbf{j} + t\mathbf{k}$, then the normal component of acceleration $\boxed{a_N} =$

(a) $2t$

(b) $\sqrt{2t}$

(c) $\frac{1}{\sqrt{2t}}$

(d) None of the above.

$|v| = \sqrt{1+2t+t^2}$

$T = \frac{\hat{i} + \sqrt{2t}\,\hat{j} + t\hat{k}}{\sqrt{1+2t+t^2}}$

$a_n = \sqrt{a^2 - a_T^2}$

$a_n = k|v|^2$

$k = \frac{|dT/dt|}{|v|}$

$a_T = \frac{d|v|}{dt}$

$a = a_T\vec{T} + a_N\vec{N}$

(14) If $\mathbf{T} = \frac{1}{\sqrt{2}}(\cos t\mathbf{i} + \sin t\mathbf{j} + \mathbf{k})$ the unit normal vector $\mathbf{N} =$ $\frac{dT/dt}{|dT/dt|}$

(a) $\mathbf{N} = \cos t\mathbf{i} + \sin t\mathbf{j}$

(b) $\mathbf{N} = \cos t\mathbf{i} - \sin t\mathbf{j}$

(c) $\mathbf{N} = -\sin t\mathbf{i} + \cos t\mathbf{j}$

(d) $\mathbf{N} = \sin t\mathbf{i} - \cos t\mathbf{j}$

$dT/dt = \frac{1}{\sqrt{2}}\left(-\sin t\,\hat{i} + \cos t\,\hat{j}\right)$

$\sqrt{\frac{1}{2}\sin^2 + \frac{1}{2}\cos^2} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

(15) $\lim_{(x,y)\to(0,0)} \frac{(x+y)^2}{x^2+y^2}$

(a) 1

(b) 0

(c) $+\infty$

(d) Does not exist.

(16) The directional derivative of the function $f(x,y) = \sqrt{2}e^{x/y}$ at the point $(0,1)$ in the direction $\mathbf{i}+\mathbf{j}$ is

(a) 1

(b) $\sqrt{2}$

(c) 2

(d) $2\sqrt{2}$

$u = \frac{1}{\sqrt{2}}\hat{i} + \frac{\hat{j}}{\sqrt{2}}$

$\nabla f = f_x i + f_y j$

$\nabla f \cdot u$

(17) The gradient of the function $f(x,y) = \ln(x^2+y^2)$ is perpendicular to the curve

(a) $x+y=1$

(b) $x^2+y^2=1$

(c) $x^2-y^2=1$

(d) $x-y=1$

$\nabla f = f_x i + f_y j + f_z k$

(18) Given that the function $f(x,y,z) = xy + y^2 + z^2$ increases most rapidly at the point $(0,1,1)$ in the direction $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$. Then

(a) $a=b=c=\frac{1}{3}$

(b) $a=b=c=\frac{2}{3}$

(c) $a=\frac{1}{3}$, $b=c=\frac{2}{3}$

(d) $a=\frac{2}{3}$, $b=c=\frac{1}{3}$

$y\hat{i} + 2y\hat{j} + 2z\hat{k}$

$\frac{1}{3}\hat{i} + \frac{2\hat{j}}{3} + \frac{2\hat{k}}{3}$

(19) Let $f(x,y) = \ln(x^2-y^2)$ then

(a) $f_{xx}+f_{yy}=0$

(b) $f_x+f_y=0$

(c) $f_{xx}-f_{yy}=0$

(d) $f_x-f_y=0$

(20) Let $f(x, y)$ be a given function and let $x = st$, $y = s^2 + t^2$, $f_x(1, 2) = 2$ and $f_y(1, 2) = -1$. Then f_s at $(s, t) = (1, 1)$ is

(a) 1

(b) 2

(c) -1

(d) 0

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s}$$
$$= (2)(1) + (-1)(2s)$$
$$2 + (-1)(2)$$
$$2 - 2$$

Question 2.(10 points) Let $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$. Find the curvature κ and the torsion τ of this curve.

$$k = \frac{|dT/dt|}{|v|}$$

$$\vec{T} = \frac{\vec{v}}{|v|}$$

$$V(t) = \hat{i} + 2t\hat{j} + 3t^2\hat{k}$$

$$|\vec{v}| = \sqrt{1 + 4t^2 + 9t^4}$$

$$\vec{T} = \frac{\hat{i} + 2t\hat{j} + 3t^2\hat{k}}{\sqrt{1 + 4t^2 + 9t^4}}$$

$$\tau = \frac{\begin{vmatrix} \dot{x} & \dot{y} & \dot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \\ \dddot{x} & \dddot{y} & \dddot{z} \end{vmatrix}}{|v \times a|^2}$$

$d\vec{T}/dt =$

$$\tau = \frac{\begin{vmatrix} 1 & 2t & 3t^2 \\ 0 & 2 & 6t \\ 0 & 0 & 6 \end{vmatrix}}{|\vec{v} \times \vec{a}|} = \frac{1 \begin{vmatrix} 2 & 6t \\ 0 & 6 \end{vmatrix}}{\sqrt{36t^4 + 36t^2 + 4}}$$

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Question 3. (10 points) Consider the function $f(x,y)=\ln(\sqrt{4-x^2-y^2})$

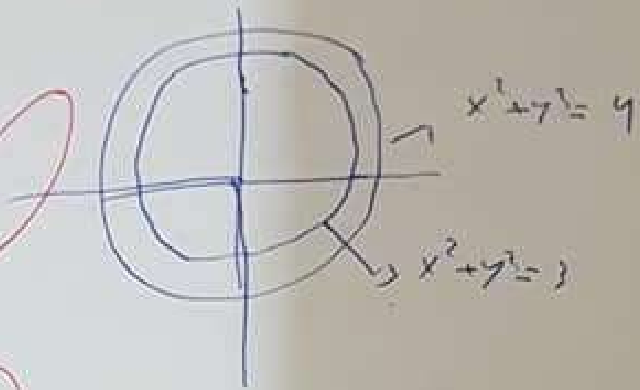
(a) Find the domain of f.

$D=\{(x,y)\mid \sqrt{4-x^2-y^2}>0\}$

(b) Find the range of f.

$[-1,1]$

(c) Draw a level curve of f.



(d) Find the boundary of the domain.

$\{(x,y)\mid x^2+y^2=4\}$

(e) Is the domain open or closed?

~~open~~ closed

(f) Is the domain bounded or unbounded?

~~un~~ bounded

(g) Find the partial derivatives of f.

$$\frac{df}{dx}=\cancel{2x}\ \frac{-x}{4-x^2-y^2}$$

$$\frac{df}{dy}=\frac{-y}{4-x^2-y^2}$$