Math 2311   November 22, 2018

Name:................   Number:..............

Q1 [6 points]. Find the **tangent plane** and **normal line** of the surface

$$\cos \pi x - x^2 y + e^{xz} + yz = 4$$

at the point $P_0(0, 1, 2)$.

Q2 [4 points]. Find the local maxima, local minima, and saddle point (if any) of the function

$$f(x, y) = e^y - y e^x.$$

Q1)

$\underline{Ans}$. let $f(x, y, z) = \cos(\pi x) - x^2 y + e^{xz} + yz - 4 = 0$

$f_x|_{P_0} = -\pi \sin(\pi x) - 2xy + z e^{xz} \big|_{P_0} = 2$

$f_y|_{P_0} = -x^2 + z \big|_{P_0} = 2$.

$f_z|_{P_0} = x e^{xz} + y \big|_{P_0} = 1$.

$\therefore \nabla f|_{P_0} = 2\vec{i} + 2\vec{j} + \vec{k}$

$\underline{Tangent\ Plane}$ $2(x-0) + 2(y-1) + 1(z-2) = 0$

or $2x + 2y + z = 4$.

$\underline{Normal\ line}$ $x = 2t,\ y = 1 + 2t,\ z = 2 + t,$
$t \in \mathbb{R}$.

Q2) $f_x = -y e^x = 0$, $f_y = e^y - e^x = 0$ $\Rightarrow (x, y) = (0, 0)$

$f_{xx} = -y e^x$, $f_{yy} = e^y$, $f_{xy} = -e^x$.

$D(0,0) = f_{xx} f_{yy} - f_{xy}^2 \big|_{(0,0)} = -y e^{x+y} - e^{2x} \big|_{(0,0)} = -1 < 0$

$\Rightarrow f$ has a saddle point at $(0, 0)$.