Birzeit University

Department of Mathematics

Quiz 6

Math 2311 — November 29, 2018

Name:.................Key — Number:..............

Important: Solve **one** of the following questions.

Q1 [10 points]. Find the point on the sphere $x^2 + y^2 + z^2 = 3$ farthest from the point $(1, -1, 1)$.

Q2 [10 points]. Find the points on the surface $z^2 = xy + 1$ nearest the origin.

Ans. Q1) let $f(x,y,z) = (x-1)^2 + (y+1)^2 + (z-1)^2$ be the square of the distance from $(1,-1,1)$.

Let $\nabla f = \lambda \nabla g$, where $g(x,y,z) = x^2+y^2+z^2-3=0$

$\Rightarrow 2(x-1)i + 2(y+1)j + 2(z-1)k = \lambda(2xi + 2yj + 2zk)$

$\Rightarrow x-1 = x\lambda \Rightarrow (1-\lambda)x = 1 \Rightarrow \boxed{x = \frac{1}{1-\lambda}}$ — (1) $\lambda \neq 1$ why?!

$2(y+1) = 2y\lambda \Rightarrow \boxed{y = \frac{1}{\lambda - 1}}$ — (2)

$z-1 = z\lambda \Rightarrow \boxed{z = \frac{1}{1-\lambda}}$ — (3)

$\boxed{x^2+y^2+z^2 = 3}$ — (4)

If we substitute (1), (2), & (3) into (4), we get

$\frac{3}{(1-\lambda)^2} = 3 \Rightarrow 1-\lambda = \pm 1 \Rightarrow$

$\Rightarrow \lambda = 1 \pm 1$

$\Rightarrow \boxed{\lambda = 2}$ or $\boxed{\lambda = 0}$

If $\lambda = 2 \Rightarrow P_1(-1,1,-1) \Rightarrow f(P_1) = 4+4+4 = 12$ abs. max.

Good Luck

If $\lambda = 0 \Rightarrow P_2(1,-1,1) \Rightarrow f(P_2) = 0$ abs. min.

$\Rightarrow$ the farthest point from $(1,-1,1)$ is $P_1(-1,1,-1)$.

(P2) let $f(x,y,z) = x^2+y^2+z^2$ be the square of the distance from the origin.

let $\nabla f = \lambda \nabla g$, where $g(x,y,z) = xy - z^2 + 1 = 0$

$$2xi + 2yj + 2zk = \lambda(yi + xj - 2zk)$$

$$\Rightarrow \begin{cases} 2x = \lambda y & \text{— (1)} \\ 2y = x\lambda & \text{— (2)} \\ 2z = -2z\lambda & \text{— (3)} \\ xy+1 = z^2 & \text{— (4)} \end{cases}$$

(1) in (2) $\Rightarrow 2y = \frac{\lambda}{2} y \cdot \lambda \Rightarrow y(4-\lambda^2) = 0$

$\Rightarrow \boxed{y=0}$ or $\boxed{\lambda = \pm 2}$

If $y = 0 \xRightarrow{eq(1)} x = 0 \xRightarrow{eq(4)} z^2 = 1 \Rightarrow z = \pm 1$

$\therefore \boxed{P_1(0,0,1)}, \boxed{P_2(0,0,-1)}$.

If $\lambda = 2$, then $\begin{cases} eq(1) \Rightarrow y = x \\ eq(3) \Rightarrow z = 0 \end{cases} \xRightarrow{eq(4)} x^2+1 = 0$

no solution.

If $\lambda = -2$, then eq (1) & eq(3) $\Rightarrow \boxed{x=-y}$ and $\boxed{z=0}$

$\xRightarrow{eq(4)} -x^2+1 = 0 \Rightarrow x = \pm 1$

If $x = 1 \Rightarrow y = -1 \Rightarrow \boxed{P_3(1,-1,0)}$.

If $x = -1 \Rightarrow y = 1 \Rightarrow \boxed{P_4(-1,1,0)}$

$f(P_1) = f(P_2) = 1$, $f(P_3) = f(P_4) = 2$

$\therefore$ P_1 and P_2 are the nearest pts to the origin.