Birzeit University

Department of Mathematics

Quiz 8

Math 2311 — December 13, 2018

Name: Key — Number:...............

Q1 [10 points]. Consider the following integral

$$I = \int_1^2 \int_0^{\sqrt{2x-x^2}} \frac{1}{(x^2+y^2)^2}\, dy\, dx.$$

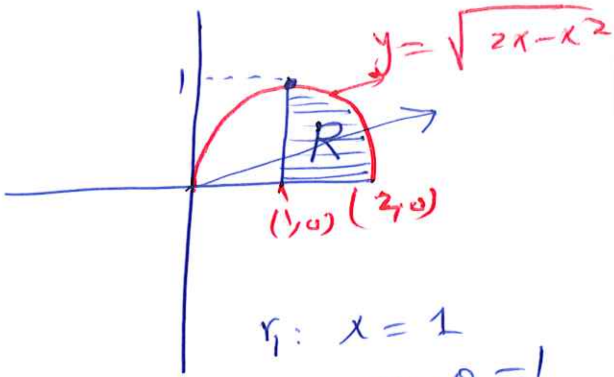
Convert the integral I into an equivalent polar integral. Then evaluate the polar integral.

Ans. $0 \le y \le \sqrt{2x - x^2}, \quad 1 \le x \le 2$

$y = 0, \quad y = \sqrt{2x - x^2}$

$x^2 - 2x + 1 + y^2 = 0 + 1 \Rightarrow (x-1)^2 + y^2 = 1$

$$I = \int_0^{\frac{\pi}{4}} \int_{\sec\theta}^{2\cos\theta} \frac{1}{(r^2)^2} \cdot r\, dr\, d\theta$$



$r_1: \; x = 1$

$r\cos\theta = 1$

$\boxed{r = \sec\theta}$

$r_2: \; y = \sqrt{2x - x^2}$

$x^2 + y^2 = 2x$

$r^2 = 2r\cos\theta$

$\Rightarrow \boxed{r = 2\cos\theta}$

$$= \int_0^{\frac{\pi}{4}} \frac{-1}{2r^2} \Big|_{\sec\theta}^{2\cos\theta} d\theta$$

$$= \int_0^{\frac{\pi}{4}} \left(\frac{-1}{8\cos^2\theta} + \frac{1}{2\sec^2\theta} \right) d\theta$$

$$= \int_0^{\frac{\pi}{4}} \left[-\frac{1}{8}\sec^2\theta + \frac{1}{2}\left(\frac{1 + \cos 2\theta}{2} \right) \right] d\theta$$

$$= \left[-\frac{1}{8}\tan\theta + \frac{1}{4}\left(\theta + \frac{\sin 2\theta}{2} \right) \right] \Big|_0^{\pi/4}$$

Good Luck

$$= -\frac{1}{8}(1) + \frac{1}{4}\left(\frac{\pi}{4} + \frac{1}{2} \right) - 0 = \frac{\pi}{16}$$

Birzeit University

Department of Mathematics

Quiz 9

Math 2311 **December 13, 2018**

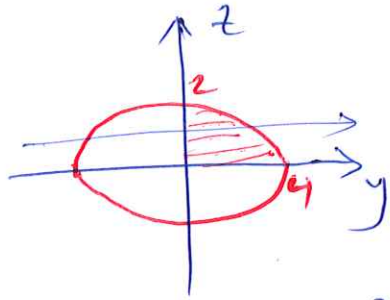
Name:........key....... Number:...............

Q1 [10 points]. Let D be the region lies in the first octant bounded by the coordinate planes, the plane $x + y = 4$ and the cylinder $y^2 + 4z^2 = 16$. Write triple integral in the order $dxdydz$ that give the volume of D. Evaluate the integral.

[**Hint:** You may use the integral:

$$\int \sqrt{4 - w^2}dw = 2\sin^{-1}\left(\frac{w}{2}\right) + \frac{1}{2}w\sqrt{4 - w^2} + C].$$

$$V = \int_0^2 \int_0^{2\sqrt{4-z^2}} \int_0^{4-y} dx\,dy\,dz$$



$$= \int_0^2 \int_0^{2\sqrt{4-z^2}} (4-y)\,dy\,dz$$

$$y^2 = 16 - 4z^2 = 4(4-z^2)$$

$$y = 2\sqrt{4-z^2}$$

$$= \int_0^2 4y - \frac{y^2}{2} \Big|_{y=0}^{y=2\sqrt{4-z^2}} dz$$

$$= \int_0^2 \left[8\sqrt{4-z^2} - 2\left(\sqrt{4-z^2}\right)^2\right] dz$$

$$= 8\int_0^2 \sqrt{4-z^2}\,dz - 2\int_0^2 (4-z^2)\,dz$$

$$= 8\left[2\sin^{-1}\frac{z}{2} + \frac{1}{2}z\sqrt{4-z^2}\right]_0^2 - 2\left(4z - \frac{z^3}{3}\right)\Big|_0^2$$

$$= 8\left[2\frac{\pi}{2} - 0\right] - 2\left(8 - \frac{8}{3} - 0\right)$$

$$= 8\pi - \frac{32}{3}$$

Good Luck