Name:.............**Key.**................   Number:..............

Question 1 Consider the curve $\mathbf{r}(t) = \cosh t\mathbf{i} - \sinh t\mathbf{j} + t\mathbf{k}$. Find the following

(a) The velocity $\mathbf{v}(t)$

① $\vec{V}(t) = \sinh t\mathbf{i} - \cosh t\mathbf{j} + \mathbf{k}$

(b) The speed $|\mathbf{v}(t)|$

① $|\vec{v}| = \sqrt{\cosh^2 t + \sinh^2 t + 1} = \sqrt{2}\,\cosh t$

(c) The acceleration $\mathbf{a}(t)$

① $\vec{a}(t) = \cosh t\mathbf{i} - \sinh t\mathbf{j} + 0\mathbf{k}$

(d) $\mathbf{v}(t) \times \mathbf{a}(t)$

②

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \sinh t & -\cosh t & 1 \\ \cosh t & -\sinh t & 0 \end{vmatrix}$$

$= \sinh t\,\hat{\mathbf{i}} + \cosh t\,\hat{\mathbf{j}} + \hat{\mathbf{k}}$

(e) $|\mathbf{v}(t) \times \mathbf{a}(t)|$

① $|\vec{v} \times \vec{a}| = \sqrt{2}\,\cosh t$

(f) The curvature $\kappa = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3} = \frac{\sqrt{2}\,\cosh t}{2\sqrt{2}\,\cosh^3 t}$

①

$= \frac{1}{2}\,\text{sech}^2 t$

(g) Find a_T and a_N.

③ $a_T = \frac{d}{dt}|\vec{v}| = \sqrt{2}\,\sinh t$

$a_N = \kappa|\vec{v}|^2 = \frac{1}{2\cosh^2 t} \cdot 2\cosh^2 t = 1$.

1

Name:......**key**.....................................   Number:................

Question 1 Consider the function $f(x,y) = \ln(x^2 + y^2 - 1)$. Find the following
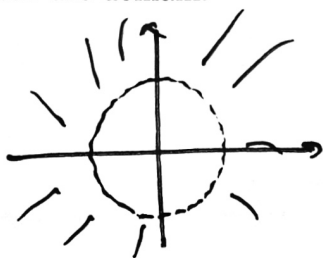
(a) Domain of f.

②

$$D = \left\{ (x,y) \mid x^2 + y^2 > 1 \right\}$$

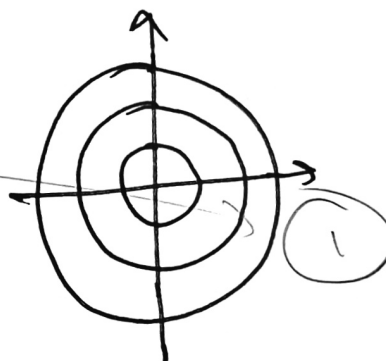(b) Range of f

②

$$(-\infty, \infty)$$

(c) Sketch the domain.

①

(d) Find the boundary of the domain.

①

$$\left\{ (x,y) \mid x^2 + y^2 = 1 \right\}$$

(e) Sketch some level curves of f.

②

$$\sqrt{\ln\left(x^2 + y^2 - 1 \right)} = C$$

① $x^2 + y^2 = e^c + 1$ Circles

(f) Is the domain open or closed.

①

open.

(g) Is the domain bounded or unbounded.

①

unbounded.

Name:................**Key.**.............   Number:..............

Question 1 Find the limit or show it does not exist

(a) $\lim\limits_{(x,y)\to(1,1)} \frac{x^4-y^4}{x-y}$

② $= \lim\limits_{(x,y)\to(1,1)} \frac{(x-y)(x+y)(x^2+y^2)}{x-y} = 4$

(b) $\lim\limits_{(x,y)\to(0,0)} \frac{x^2y}{x^2+y^2}$

② $= \lim\limits_{r\to 0} \frac{r^3 \cos^2\theta \sin\theta}{r^2} = 0$

(c) $\lim\limits_{(x,y)\to(0,0)} \frac{x-y}{x+y}$ $y=mx$

② $= \lim\limits_{(x,y)\to(0,0)} \frac{x(1-m)}{x(1+m)} = \frac{1-m}{1+m}$ DNE

Question 2 Let $f(x,y) = \tan(xy)$. Find $f_x, f_y, f_{xx}, f_{yy}, f_{xy}$.

② $f_x = y \sec^2(xy)$, $f_y = x\sec^2(xy)$ ①

① $f_{xx} = 2y^2 \sec^2(xy)\tan(xy)$

① $f_{yy} = 2x^2 \sec^2(xy)\tan(xy)$

① $f_{xy} = \sec^2(xy) + 2xy\sec^2(xy)\tan(xy)$.