Berzeit University

Department of Mathematics

Quiz 2

Math 2311 October 11, 2018

Name:.................. Number:...............

Q1 [5 pts]. Find the point on the curve $\mathbf{r}(t) = (12\sin t)\ \mathbf{i} - (12\cos t)\mathbf{j} + 5t\mathbf{k}$ at a distance 13π units along the curve from the point $(0, -12, 0)$ in the direction opposite to the direction of increasing arc length.

Q2 [5 pts]. A particle moves in the plane so that its velocity and position vectors are always orthogonal. Show that the particle moves in a circle centered at the origin.

Q1) $\vec{v}(t) = \frac{d\vec{r}}{dt} = (12\cos t)\, i + (12\sin t)\, j + 5k$

$s(t) = \int_0^t |v(\tau)|\, d\tau$

$-13\pi = \int_0^t \sqrt{144\cos^2\tau + 144\sin^2\tau + 25}\, d\tau$

$\Rightarrow -13\pi = 13t \Rightarrow \boxed{t = -\pi}$

$\therefore$ point $(0, +12, -5\pi)$.

Q2) Let $\vec{r}(t)$ be the position vector.

Given that $\vec{r}(t) \cdot \frac{d\vec{r}}{dt} = 0$.

$\Rightarrow \frac{d}{dt}|\vec{r}|^2 = \frac{d}{dt}(\vec{r}\cdot\vec{r}) = 2\vec{r}\cdot\frac{d\vec{r}}{dt} = 0$.

$\Rightarrow |\vec{r}|^2 = $ Constant (say c).

$\Rightarrow \boxed{x^2 + y^2 = c}$

Good Luck

Key

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Quiz 3

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Q1[4 pts]. Find the point on the curve $y = e^x$ where the curvature is greatest.

Q2[3 pts]. Find an equation for the **osculating plane** of the curve $\mathbf{r}(t) = t\mathbf{i}+t^2\mathbf{j}+t^3\mathbf{k}$ at $t = 1$.

Q3[3 pts]. **Without finding T** and **N**, write the acceleration of the motion $\mathbf{r}(t) = (4\cos t)\ \mathbf{i}+(\sqrt{2}\sin t)\mathbf{j}$ in the form $\mathbf{a}=a_T\ \mathbf{T}+a_N\mathbf{N}$ at $t = \frac{\pi}{4}$.

Ans. Q1) let $x=t \Rightarrow y = e^t$.

$$\boxed{\vec{r}(t) = t\,i + e^t j}$$

$$\vec{V}(t) = i + e^t j, \quad \vec{a}(t) = e^t j$$

$$\vec{V}\times\vec{a} = \begin{vmatrix} i & j & k \\ 1 & e^t & 0 \\ 0 & e^t & 0 \end{vmatrix} = e^t k.$$

$$\Rightarrow |\vec{V}\times\vec{a}| = e^t, \quad |\vec{V}| = \sqrt{1+e^{2t}}.$$

$$\therefore \kappa(t) = \frac{|\vec{V}\times\vec{a}|}{|\vec{V}|^3} = \frac{e^t}{(1+e^{2t})^{3/2}}$$

$$\kappa'(t) = \frac{e^t(1-2e^{2t})}{(1+e^{2t})^{5/2}}. \quad \text{check!!}$$

$$= 0 \text{ if } \boxed{t = -\tfrac{1}{2}\ln 2}.$$

check!!

κ': $+++$ $---$ at $-\frac{1}{2}\ln 2$

Good Luck

$\Rightarrow$ point: $x = -\frac{1}{2}\ln 2$

$y = e^{-\frac{1}{2}\ln 2} = \frac{1}{\sqrt{2}} \Rightarrow$ point $\left(-\frac{1}{2}\ln 2, \frac{1}{\sqrt{2}}\right)$.

Q2) $r(t) = ti + t^2 j + t^3 k$, $t = 1$

$P(1,1,1)$, $\vec{V}(t) = i + 2tj + 3t^2 k$.

$V(1) = i + 2j + 3k$, $|V(1)| = \sqrt{14}$

$$\therefore T(1) = \frac{V(1)}{|V(1)|} = \frac{1}{\sqrt{14}}(i + 2j + 3k).$$

$$T(t) = \frac{1}{\sqrt{14}}(i + 2tj + 3t^2 k)$$

$$\frac{dT}{dt} = \frac{1}{\sqrt{14}}(2j + 6tk).$$

$$\left.\frac{dT}{dt}\right|_{t=1} = \frac{1}{\sqrt{14}}(2j + 6k).$$

$$\left|\frac{dT}{dt}\right| = \frac{1}{\sqrt{14}}\left(\sqrt{4+36}\right) = \frac{2\sqrt{10}}{\sqrt{14}}.$$

$$\therefore N(1) = \frac{dT}{dt} \Big/ \left|\frac{dT}{dt}\right| = \frac{1}{2\sqrt{10}}(2j + 6k).$$

$$B(1) = T(1) \times N(1) = \begin{vmatrix} i & j & k \\ \frac{1}{\sqrt{14}} & \frac{2}{\sqrt{14}} & \frac{3}{\sqrt{14}} \\ 0 & \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \end{vmatrix}$$

$$= \frac{3}{\sqrt{140}} i - \frac{3}{\sqrt{140}} j + \frac{1}{\sqrt{140}} k.$$

eq: $\frac{3}{\sqrt{140}}(x-1) \cancel{+} - \frac{3}{\sqrt{140}}(y-1) + \frac{1}{\sqrt{140}}(z-1) = 0.$

or $\boxed{3x - 3y + z = 1}$ is an eq. of the osculating plane.

Q3) $\vec{r}(t) = (4\cos t)i + (\sqrt{2}\sin t)j \quad , t = \frac{\pi}{4}.$

$$\vec{V}(t) = -(4\sin t)i + (\sqrt{2}\cos t)j$$

$$\vec{a}(t) = -(4\cos t)i - (\sqrt{2}\sin t)j$$

$$a_T = \frac{d}{dt}|\vec{V}| = \frac{d}{dt}\left(\sqrt{16\sin^2 t + 2\cos^2 t}\right)$$

$$a_T \Big|_{t=\frac{\pi}{4}} = \frac{32\sin t\cos t \cancel{+} 4\cos t\sin t}{2\sqrt{16\sin^2 t + 2\cos^2 t}}$$

$$= \frac{32\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2} - 4\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}}{2\sqrt{8+1}} = 7/3.$$

$$|\vec{a}(\tfrac{\pi}{4})|^2 = \left(-4\frac{\sqrt{2}}{2}\right)^2 + \left(-\sqrt{2}\cdot\frac{\sqrt{2}}{2}\right)^2 = 9.$$

$$\therefore \quad a_N = \sqrt{|a|^2 - a_T^2} = \sqrt{9 - \frac{49}{9}} = \frac{4\sqrt{2}}{3}.$$