Berzeit University

Department of Mathematics

Quiz 4

Math 2311 **October 30, 2018**

Name:......Key...... **Number:**...............

Q1 [4 pts]. Let $f(x,y) = \frac{\sqrt{x-y^2}}{\ln(4-x^2-y^2)}$.

(a) Find and sketch the domain of f.
(b) Is the domain of f bounded?
(c) Is the domain of f open, closed or neither?

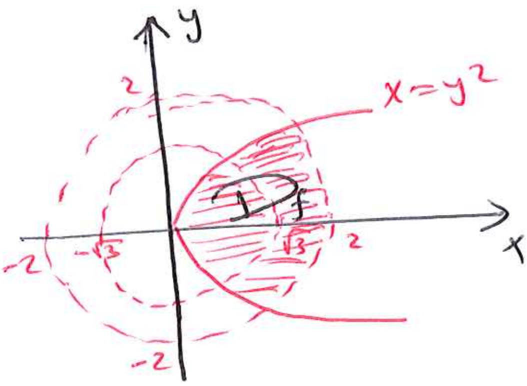
Q2 [4 pts]. Find the limit, if it exists, or show that the limit does not exist.

(a) $\lim_{(x,y)\to(1,0)} \frac{xy-y}{(x-1)^2+y^2}$.

(b) $\lim_{(x,y)\to(0,0)} \frac{x^{2018}\sin^2 y}{x^{2018}+2y^{2016}}$.

Q3 [2 pts]. Let $z = (xy)^x$. Find $\frac{\partial^2 z}{\partial y \partial x}\big|_{(1,2)}$.

Ans. Q1) a) $D_f = \{(x,y) \mid x \geq y^2,\ x^2+y^2 < 4, \text{ and } x^2+y^2 \neq 3\}$.



b) bounded. Since it lies in a disk of radius $r > 2$. and centered at $(0,0)$.

c) D_f is neither open nor closed. Since the region doesn't include all its boundary and not all its points are interiors.

Good Luck

Q2) (a). $\displaystyle\lim_{(x,y)\to(1,0)} \frac{xy-y}{(x-1)^2+y^2}$

along $y=x-1$

$$= \lim_{x\to 1} \frac{x(x-1)-x+1}{(x-1)^2+(x-1)^2} = \lim_{x\to 1} \frac{(x-1)^2}{2(x-1)^2} = \frac{1}{2}$$

• $\displaystyle\lim_{(x,y)\to(1,0)} \frac{xy-y}{(x-1)^2+y^2}$

along $y=1-x$

$$= \lim_{x\to 1} \frac{x(1-x)+x-1}{(x-1)^2+(1-x)^2} = \lim_{x\to 1} \frac{-(x-1)^2}{2(x-1)^2} = -\frac{1}{2}$$

$\therefore$ Two-path test $\Rightarrow$ lim DNE.

(b) For $(x,y) \neq (0,0)$, we have $\boxed{0 \leq \frac{x^{2018}}{x^{2018}+2y^{2016}} \leq 1}$

$$\Rightarrow 0 \leq \frac{x^{2018}\sin^2 y}{x^{2018}+2y^{2016}} \leq 1 \cdot \sin^2 y.$$

Since $\displaystyle\lim_{(x,y)\to(0,0)} 0 = \lim_{(x,y)\to(0,0)} \sin^2 y = 0$ then by

the squeeze Thm, $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^{2018}\sin^2 y}{x^{2018}+2y^{2016}} = 0.$

Q3) $z = (xy)^x \Rightarrow \ln z = x \ln(xy)$

$$\Rightarrow \boxed{x\ln(xy) - \ln z = 0}.$$

let $\boxed{F(x,y,z) = x\ln(xy) - \ln z}$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{\ln(xy) + x\cdot\frac{y}{xy}}{-\frac{1}{z}}$$

$$\Rightarrow \boxed{\frac{\partial z}{\partial x} = (xy)^x\left[1 + \ln(xy)\right]}$$

Now, $\dfrac{\partial^2 z}{\partial y \partial x} = Z_{xy}$

$$= \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$= \frac{\partial}{\partial y}\left((xy)^x\left[1 + \ln(xy)\right]\right)$$

$$= y^x x^x\left(\frac{1}{xy}\cdot x\right) + \left(1 + \ln(xy)\right)\cdot x^x \cdot x y^{x-1}$$

$$= \frac{(xy)^x}{y} + x^2(xy)^{x-1}(1 + \ln xy)$$

$$\therefore \left.\frac{\partial^2 z}{\partial y \partial x}\right|_{(1,2)} = \frac{2^1}{2} + 1(2)^0(1 + \ln 2)$$

$$= 2 + \ln 2.$$