

Birzeit University  
Department of Mathematics

Quiz 7

Math 2311

December 6, 2018

Name:.....*Key*.....

Number:.....

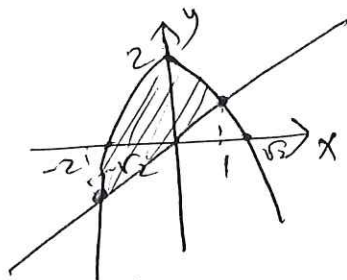
**Q1 [3 points].** Set up an integral that give the volume of the solid that is bounded above by the cylinder  $z = x^2$  and below by the region enclosed by the parabola  $y = 2 - x^2$  and the line  $y = x$  in the  $xy$ -plane.

**Q2 [7 points].** Consider the following integral

$$I = \int_0^{1/16} \int_{y^{1/4}}^{1/2} \cos(16\pi x^5) dx dy.$$

- (a) Sketch the region of integration.  
 (b) Write an equivalent double integral with the order of integration reversed.  
 (c) Evaluate the integral.

*Ans. Q1)  
(3 pts)*



$$2 - x^2 = x \Rightarrow x^2 + x - 2 = 0$$

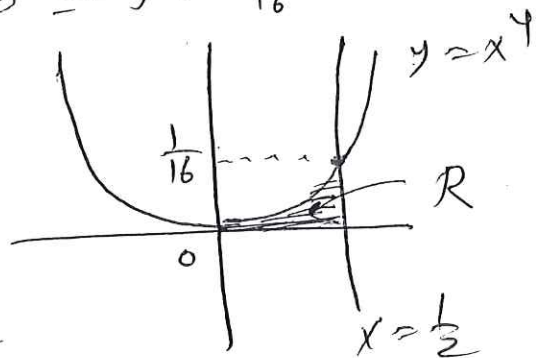
$$(x+2)(x-1) = 0$$

$$x = -2, x = 1$$

$$V = \int_{-2}^1 \int_x^{2-x^2} x^2 dy dx$$

Q2) (a)  $y^{1/4} \leq x \leq \frac{1}{2}, \quad 0 \leq y \leq \frac{1}{16}$

(2 pts)  $x = y^{1/4} \Rightarrow y = x^4$



(b)  $I = \int_0^{1/2} \int_0^{x^4} \cos(16\pi x^5) dy dx$

(2 pts)

Good Luck

c) Let  $I_1 = \int_0^{x^4} \cos(16\pi x^5) dy$  (2 pts)  $= y \cos(16\pi x^5) \Big|_{y=0}^{y=x^4}$

$$\Rightarrow I_1 = x^4 \cos(16\pi x^5)$$

$$\therefore I = \int_0^{\frac{1}{2}} x^4 \cos(16\pi x^5) dx$$

$$\text{let } \boxed{u = 16\pi x^5} \Rightarrow du = 80\pi x^4 dx$$
$$\Rightarrow \boxed{\frac{1}{80\pi} du = x^4 dx}$$

$$0 \leq x \leq \frac{1}{2} \Rightarrow 0 \leq u \leq \frac{\pi}{2}$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \cos(u) \frac{du}{80\pi}$$

$$= \frac{1}{80\pi} \sin u \Big|_0^{\frac{\pi}{2}} = \frac{1}{80\pi} (1 - 0)$$
$$= \frac{1}{80\pi}$$

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Good Luck.

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