Math 231

Birzeit University
Mathematics Department
Final Test

Math 231 Calculus III
Dec 15, 2009

First Semester 2009/2010

(KEY)

Student Name: ______________________
Student Number: ______________________

Instructor: (Check only one box)
☐ Abdul-Hamid Aburrub ☐ Aalaa Armiti ☐ Shadi Omari

19 × 3

57

Question # 1 (57 %)
Circle the letter that corresponds to the best answer for each question:

Use the information in the box to answer questions 1 – 3

Suppose that $f(x,y)$ is a differentiable function satisfying $f(1,3)=1,\ \ f_x(1,3)=2,\ \ f_y(1,3)=4,$

(1) Find a vector that is perpendicular to the level curve $f(x,y)=1$ at the point $(1,3)$.

(a) $2\hat{i}+2\hat{j}$ (b) $2\hat{i}-\hat{j}$ (c) $-\hat{i}+4\hat{j}$ (d) $\hat{i}+2\hat{j}$

(2) Find the derivative of the function $z=f(x,y)$ at the point $(1,3)$ in the direction of $-\hat{i}+2\hat{j}$.

(a) $\frac{3}{\sqrt{5}}$ (b) $\frac{6}{\sqrt{5}}$ (c) $\sqrt{5}$ (d) $2\sqrt{5}$

(3) Use linear approximation to estimate the value of $f(1.2, 3.1)$.

(a) 2.1 (b) 0.4 (c) 1.8 (d) 1.4

(4) Find the volume of the solid region W, in the first octant, bounded from above by the plane $z=x+y$, and from the sides by the cylinder: $x^2+y^2=4$, and from below by the xy-plane.

(a) $\frac{16}{3}$ (b) 8 (c) 16 (d) $\frac{4}{3}$

Use the information in the box to answer questions 5 – 6

Suppose that the integral of a function over a region R is given in polar coordinates by $\int_0^3 \int_0^{\pi/2} r^2 d\theta\, dr$

(5) Convert the integral to Cartesian coordinates

(a) $\int_0^3 \int_0^{\sqrt{9-x^2}} \sqrt{x^2+y^2}\, dy\, dx$ (b) $\int_0^3 \int_0^{\sqrt{9-x^2}} \left(x^2+y^2\right) dy\, dx$

(c) $\int_0^3 \int_0^3 \sqrt{x^2+y^2}\, dx\, dy$ (d) $\int_0^3 \int_0^{\sqrt{9-x^2}} \left(x^2+y^2\right) dx\, dy$

(6) Evaluate the integral, (in polar or in Cartesian).

(a) $\left(\frac{\pi}{2}\right)^3$ (b) 9π (c) $\frac{9\pi}{2}$ (d) 27π

Consider the ellipsoid $x^2+y^2+2z^2=4$ and the point $P(1,1,1)$ to answer questions 7 – 8

(7) Find parametric equations for the line that is normal to the ellipsoid at P.

(a) $x=1+t,\ y=1+t,\ z=1+2t$ (b) $x=1+2t,\ y=1+2t,\ z=1+2t$

(c) $x=2+t,\ y=2+2t,\ z=2+4t$ (d) $x=1+2t,\ y=1+t,\ z=1+4t$

(8) The line in question 7 intersects the ellipsoid in another point. Find that point.

(a) $\left(\frac{11}{5},\frac{11}{5},\frac{17}{5}\right)$ (b) $\left(\frac{11}{5},\frac{11}{5},\frac{-7}{5}\right)$ (c) $\left(\frac{-1}{5},\frac{-1}{5},\frac{17}{5}\right)$ (d) $\left(\frac{-1}{5},\frac{-1}{5},\frac{-7}{5}\right)$

(9) Compute the integral by reversing the order of integration

$$\int_0^1 \int_{\sqrt{y}}^1 \frac{y\, e^{x^2}}{x^3}\, dx\, dy$$

(a) $e-1$ (b) $\frac{e-1}{4}$ (c) $\frac{e^2-1}{4}$ (d) $\frac{e-1}{2}$

(10) If $f(x,y,z)=\left[z^2+z\sin(x-3y)\right]^2$, then compute $\frac{\partial f}{\partial x}$ at the point $\left(0,\frac{\pi}{3},1\right)$

(a) -1 (b) 1 (c) 2 (d) -2

(11) If $f(x,y,z)=x^2+2y-yz$, $\bar{u}=2\hat{i}+2\hat{j}-\hat{k}$ and $P(1,-2,1)$, then find $(D_{\bar{u}}f)_P$

(a) $\frac{13}{3}$ (b) $\frac{4}{3}$ (c) $\frac{11}{3}$ (d) $-\frac{1}{3}$

(12) Evaluate the integral in cylindrical coordinates

$$\int_0^{2\pi}\int_0^1\int_0^{\sqrt{1-r^2}} zr\, dz\, dr\, d\theta$$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{8}$ (d) $\frac{\pi}{4}$

(13) Find the limit

$$\lim_{(x,y)\to(0,0)}\left[\frac{2xy}{x^2+2y^2}\right]$$

(a) 0 (b) 1 (c) ∞ (d) does not exist

Use the information in the box to answer questions 14 – 16

The position vector of a particle moving in the space is given by $$\bar{r}(t) = (2\cos t)\,\hat{i} + (\sin t)\,\hat{j} + t\,\hat{k}$$

(14) Find the velocity at $t = 0$

(a) $\hat{i}+\hat{j}$ (b) $\hat{j}+\hat{k}$ (c) $2\hat{i}$ (d) $-2\hat{i}$

(15) Find the acceleration at $t = 0$

(a) $\hat{i}+\hat{j}$ (b) $\hat{j}+\hat{k}$ (c) $2\hat{i}$ (d) $-2\hat{i}$

(16) Find the curvature at $t = 0$

(a) 2 (b) $\sqrt{2}$ (c) 1 (d) $\frac{1}{2}\sqrt{2}$

(17) Find the volume, (use spherical coordinates), of the solid described by

$D = \{(x, y, z) | x^2 + y^2 + z^2 \le 4, x \le 0, y \ge 0, z \le 0\}$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{8\pi}{3}$ (d) $\frac{4\pi}{3}$

(18) Find a local maximum of the function $f(x, y) = y$ on the curve $x^2 + xy + y^2 = 3$

(a) 0 (b) 1 (c) 2 (d) -1

(19) If $w = \frac{1}{2x+y}$, and $x = t$, $y = -t$, find $\frac{dw}{dt}$ at the point $(1, -1)$.

(a) -1 (b) 1 (c) 2 (d) -2

Question # 2 14%

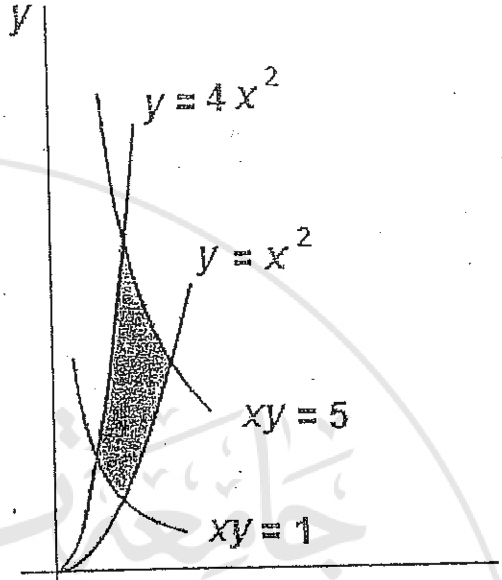
Evaluate the integral $\iint_R xy\, dA$ over the region bounded by the curves

$xy = 1, xy = 5, y = x^2$, and $y = 4x^2$

as in the figure

Use the transformation

$u = xy, \; v = x^2$



The Transformation

2 $u = xy \; , \; v = x^2$

The Inverse ⟹ u

2 $x = \sqrt{v} \; , \; y = \frac{u}{\sqrt{v}}$

The Jacobian

2 $J = \begin{vmatrix} 0 & \frac{1}{2} v^{-\frac{1}{2}} \\ v^{-\frac{1}{2}} & -u\frac{1}{2} v^{-\frac{3}{2}} \end{vmatrix} = -\frac{1}{2} v^{-1} = \frac{-1}{2v}$

The region in the uv-plane

4 $xy = 1 \Longrightarrow u = 1 \; , \; xy = 5 \Longrightarrow u = 5$

$y = x^2 \Longrightarrow \frac{u}{\sqrt{v}} = v \Longrightarrow u = v^{3/2} \Rightarrow v = u^{2/3}$

$y = 5x^2 \Longrightarrow u = 5v^{3/2} \Rightarrow v = \left(\frac{u}{4}\right)^{2/3}$

The integral

3 $\int_1^5 \int_{u^{2/3}}^{(u/4)^{2/3}} \sqrt{v} \cdot \frac{u}{\sqrt{v}} \cdot \frac{1}{2v} \, \cancel{du\, dv}$ $dv\, du$. $\int_1^5 \int_{(u/4)^{2/3}}^{u^{2/3}} \frac{u}{2v} \, dv\, du$

Evaluation $\int \frac{1}{2v} \left[\frac{u^2}{2}\right]_{v^{3/2}}^{5v^{3/2}} dv$

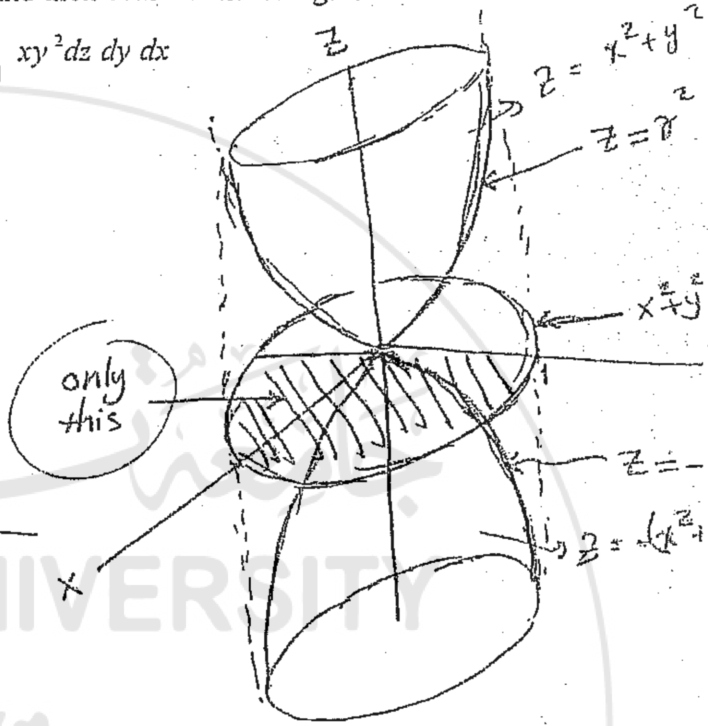
1 $= \int_1^5 6v^2 \, dv = 248$

14

Question # 3 (14%)

Change the integral to cylindrical coordinate, and then evaluate the integral

$$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-(x^2+y^2)}^{(x^2+y^2)} xy^2\,dz\,dy\,dx$$



The region

$-r^2 \le z \le r^2$

$-\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$

$0 \le x \le 1$

The integral

8 $\int\int\int xy^2\,dz\,dy\,dx$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^1 \int_{-r^2}^{r^2} r\cos\theta\,(r\sin\theta)^2\,dz\;r\,dr\,d\theta$$

Evaluation

6 $\int\int\int_{-r^2}^{r^2} r^4\cos\theta\,\sin^2\theta\,dz\,dr\,d\theta$

$$= \int\int_0^1 2r^6\cos\theta\,\sin^2\theta\,dr\,d\theta \qquad \frac{r^7}{7}$$

$$= \int \frac{2}{7}\cos\theta\,\sin^2\theta\,d\theta \qquad \text{let } u = \sin \Rightarrow du = \cos$$

2

$$= \frac{2}{7}\,\frac{\sin^3\theta}{3}\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{4}{21}$$

14

Question # 4 (16%)

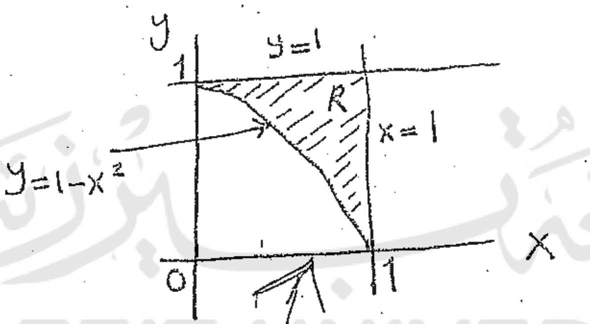
Find the average value of the function $f(x, y) = 2xy$ over the region R enclosed by the curves and lines: $y = 1 - x^2$, $y = 1$, and $x = 1$

2

av $=$ Average Value $= \dfrac{1}{\text{Area}(R)} \iint_R f(x,y)\,dA$

The Region R

4



4 Area $(R) = \displaystyle\int_0^1 \int_{1-x^2}^{1} \boxed{dy\,dx}$

2 Value of the area $= \int [1-(1-x^2)]\,dx$

$= \int_0^1 x^2\,dx = \frac{1}{3}$

The integral

3 $\iint f(x,y)\,dA = \displaystyle\int_0^1 \int_{1-x^2}^{1} 2xy\,dy\,dx$

$= \int x\left[1-(1-x^2)^2\right]dx$

$= \int_0^1 (2x^3 - x^5)\,dx$

$= \frac{2}{4} - \frac{1}{6}$

$= \frac{1}{3}$

1 av $=$ $\frac{1}{1/3} \cdot \frac{1}{3} = 1$ 16

صالح

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(2) Find the derivative of the function $z=f(x,y)$ at the point $(1,3)$ in the direction of $-\hat{\text{i}}+2\hat{\text{j}}$

(a) $\dfrac{3}{\sqrt{5}}$ (b) $\dfrac{6}{\sqrt{5}}$ (c) $\sqrt{5}$ (d) $2\sqrt{5}$

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$$\lim_{(x,y)\to(0,0)}\left[\frac{2xy}{x^2+2y^2}\right]$$

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Question # 2 14%

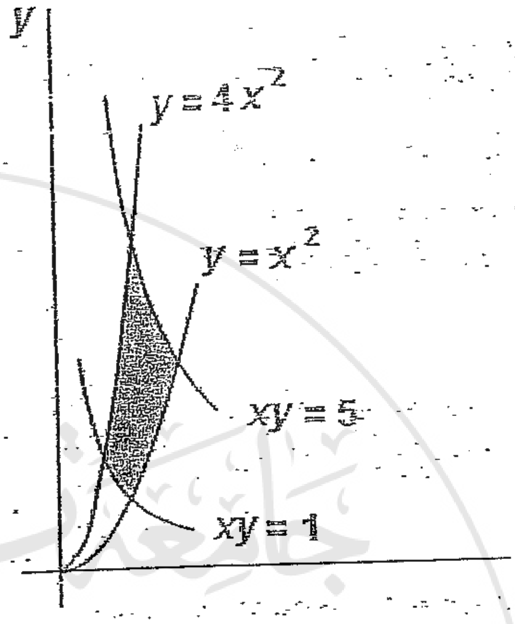
Evaluate the integral $\iint_R xy\, dA$ over the region bounded by the curves

$xy = 1$, $xy = 5$, $y = x^2$, and $y = 4x^2$

as in the figure

Use the transformation

$u = xy$, $v = x^2$



The Transformation

2 $u = xy \,,\, v = x^2$

The Inverse

2 $x = \sqrt{v} \,,\, y = \dfrac{u}{\sqrt{v}}$

The Jacobian

2 $J = \begin{vmatrix} 0 & \frac{1}{2} v^{-\frac{1}{2}} \\ v^{-\frac{1}{2}} & -u \frac{1}{2} v^{-\frac{3}{2}} \end{vmatrix} = -\frac{1}{2} v^{-1} = \frac{-1}{2}$

The region in the uv-plane

4 $xy = 1 \Longrightarrow u = 1 \quad , \quad xy = 5 \Longrightarrow u = 5$

$y = x^2 \Longrightarrow \dfrac{u}{\sqrt{v}} = v \Longrightarrow u = v^{3/2} \Rightarrow v = u^{2/3}$

$u = 5v^{3/2} \Rightarrow v = \left(\dfrac{u}{4}\right)^{2/3}$

$y = 5x^2$

The integral

3 $\displaystyle\int_1^5 \int_{u^{2/3}}^{5v^{3/2}\,(\frac{u}{4})^{2/3}} \sqrt{v} \cdot \frac{u}{\sqrt{v}} \cdot \frac{1}{2v} \, du\, dv \quad dv\, du \quad \int_1^5 \int_{(\frac{u}{4})^{2/3}}^{u^{2/3}} \frac{u}{2v} \, dv\, du$

Evaluation

1 $\displaystyle\int \frac{1}{2v} \left[\frac{u^2}{2}\right]_{v^{3/2}}^{5v^{3/2}} dv$

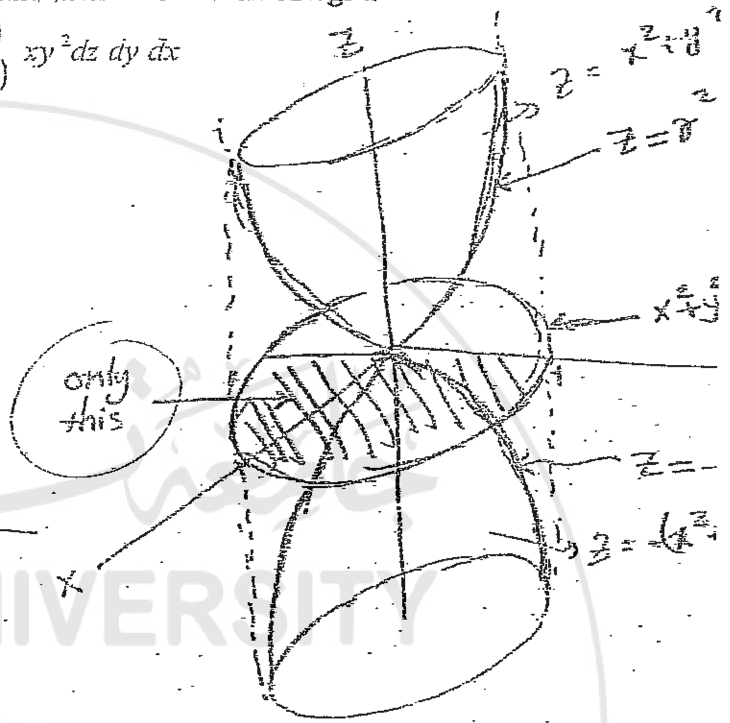
$\displaystyle = \int_1^5 6v^2\, dv \quad = 248$

14

Question # 3 (14%)

Change the integral to cylindrical coordinate, and then evaluate the integral

$$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-(x^2+y^2)}^{(x^2+y^2)} xy^2 \, dz \, dy \, dx$$



The region

$-r^2 \le Z \le r^2$

$-\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$

$0 \le x \le 1$

The integral

$$\int\int\int xy^2 \, dz\, dy\, dx$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^1 \int_{-r^2}^{r^2} r\cos\theta \, (r\sin\theta)^2 \, dz \; r\, dr\, d\theta$$

Evaluation

$$\int\int\int_{-r^2}^{r^2} r^4 \cos\theta \sin^2\theta \, dz\, dr\, d\theta$$

$$= \int\int_0^1 2r^6 \cos\theta \sin^2\theta \, dr\, d\theta \qquad \frac{r^7}{7}$$

let $u = \sin \quad du = \cos$

$$= \int \frac{2}{7} \cos\theta \sin^2\theta \, d\theta$$

$$= \frac{2}{7} \left. \frac{\sin^3\theta}{3} \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{4}{21}$$

14

Question # 4 (16%)

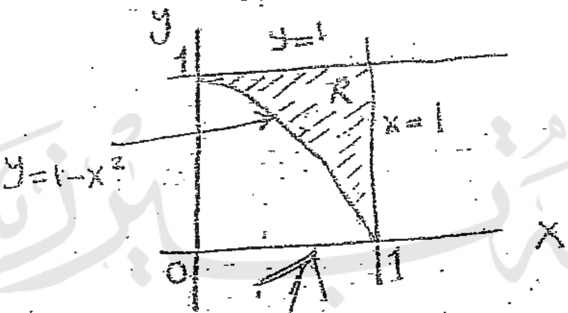
Find the average value of the function $f(x, y) = 2xy$ over the region R enclosed by the curves and lines: $y = 1 - x^2$, $y = 1$, and $x = 1$

2

$av =$ Average Value $= \dfrac{1}{\text{Area}(R)} \iint_R f(x,y)\,dA$

The Region R

4



4

$\text{Area}(R) = \int_0^1 \int_{1-x^2}^{1} \boxed{dy\,dx}$

Value of the area $= \int [1-(1-x^2)]\,dx$

2

$= \int_0^1 x^2\,dx = \frac{1}{3}$

The integral

$\iint f(x,y)\,dA = \int_0^1 \int_{1-x^2}^{1} 2xy\,dy\,dx$

3

$= \int x\left[1-(1-x^2)^2\right]dx$

$= \int_0^1 (2x^3 - x^5)\,dx$

$= \frac{2}{4} - \frac{1}{6}$

$= \frac{1}{3}$

1 $av =$ $\frac{1}{1/3} \cdot \frac{1}{3} = 1$

16

$24(2) = 18 + 6 + 8 + 15 + 10 = 85$

Birzeit University
Mathematics Department
Math 231 - Final Exam
Instructors: Dr. Khaled Al-Takhman, Mr. Rasim Kaabi

Summer 2006

Student Name: ....Asmaa Nael Arar.......... Number:.1050853..Section:.......

D.r Rasem Kaabi

Question 1 (54%). Circle the most correct answer:

1. If a region R in the plane is symmetric about the x-axis and the y-axis, then

   (a) $\int\int_R f(x,y)\ ; dxdy = 2\int\int_G f(x,y)\ dxdy$, G is the part of R in the first quadrant.

   (b) $\int\int_R f(x,y)\ ; dxdy = 4\int\int_G f(x,y)\ dxdy$, G is the part of R in the first quadrant.

   (c) $\int\int_R f(x,y)\ ; dxdy = 4\int\int_G f(x,y)\ dxdy$, G is the part of R in the first and second quadrants.

2. An equation of the line that passes through the point $(-8,-3)$ perpendicular to $\vec{v} = -5i+4j$ is

   $-5(x+8)+4(y+3)=0$

   $-5x-40+4y+12=0$

   $-5x+4y=28$

   (a) $-5x+4y=28$.

   (b) $y+3 = \frac{-4}{5}(x+8)$.

   (c) $-4x+5y=17$.

   (d) $-5x+4y=41$.

   $t-6=s$
   $t=s$

3. The lines $L_1 : x = t-6, y = t, z = 2t$ and $L_2 : x = t, y = t, z = -t$

   $\langle 1,1,2\rangle$
   $\langle 1,1,-1\rangle$

   not parallel

   $t_1 - 6 = t_2$
   $t_1 = t_2$
   $0-6=$

   (a) intersect at a point

   (b) are parallel

   (c) not parallel and do not have intersection point

   (d) none.

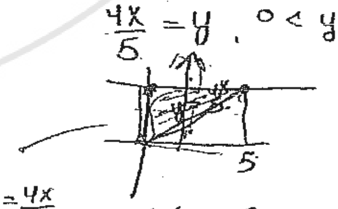
   $1-6$

   $t-6=t_1$
   $t=t_1$

4. When reversing the order of integration of $\int_0^4\int_0^{\frac{5y}{4}} dxdy$ we get

   $x = \frac{5}{4}y$

   $\frac{4x}{5} = y$, $0 < y < 4$

   (a) $\int_0^5\int_0^{\frac{5x}{4}} dydx$

   (b) $\int_0^5\int_0^{\frac{x}{4}} dydx$

   (c) $\int_0^5\int_{\frac{4x}{5}}^4 dydx$

   (d) $\int_0^{\frac{5y}{4}}\int_0^4 dydx$

   $x=$

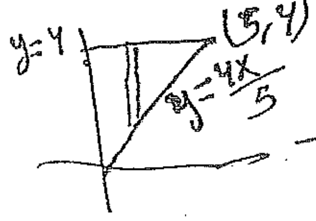
   $y=0$
   $y=4$

   $\int_0^5\int_{y=0}^{y=\frac{4x}{5}} dy\,dx$

   $4 = \frac{4}{5}x$
   $x=5$

   $x=5$

   $\int_{x=0}\int_{y=\frac{4x}{5}}^{y=4}$

$$\int_1^{e^2}\int^{e^8}\int_1^{e^9} \frac{1}{xyz}\,dx\,dy\,dz$$

$$\frac{\ln x}{yz}\Big|_1^{e^9}$$

$$\Rightarrow \frac{9}{yz} - \frac{1}{yz} \Rightarrow \frac{8}{yz} \Rightarrow \frac{8\ln y}{z}$$

$$\frac{8(8)}{z} - \frac{8}{z}$$

$$\begin{array}{r} {}^{5}\cancel{6}4 - \\ 18 \\ \hline 46 \end{array}$$

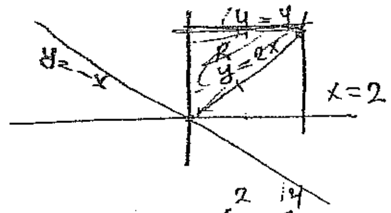
$$\frac{64-8}{z} \Rightarrow$$

$$\frac{46}{z} \Rightarrow 46\ln z$$

$$46\times 2 - 46$$

$$\Rightarrow 46$$

5. $\int_0^4 \int_{\frac{y}{2}}^{2} f(x,y)\,dxdy + \int_{-2}^{0} \int_{-y}^{2} f(x,y)\,dxdy =$



$2x = y$

(a) $\int_0^2 \int_{-x}^{2x} f(x,y)\,dydx$

$\int_0^2 \int_{2x}^{4} f(x,y)\,dydx$

(b) $\int_0^4 \int_{-2x}^{x} f(x,y)\,dydx$

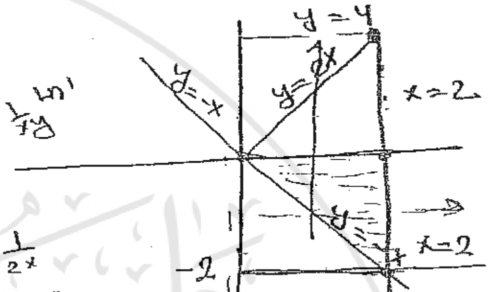
$x = 2$
$y = -x$

(c) $\int_0^2 \int_{-x}^{x} f(x,y)\,dydx$

$x =$

$x = 2$
$2x =$

(d) $\int_0^2 \int_{-2}^{4} f(x,y)\,dydx$



6. $\int_{-7}^{7} \int_{-\sqrt{49-y^2}}^{\sqrt{49-y^2}} dxdy =$ زاوية

$\int_0^{2\pi} \int_0^7 r.drd\theta$

$\int_0^{2\pi} \frac{r^2}{2}.d\theta \Big|_0^7$

$\int_0^{2\pi} \frac{49}{2}\theta$ $\quad \frac{49}{2}(2\pi)$ $\quad 49\pi$

(a) 7π

(b) 49π

(c) 196π

(d) 98π

$\int_{y=-x}^{y=2x} \int_0^2 dxd$

7. $\int_1^{e^2} \int_1^{e^8} \int_1^{e^9} \frac{1}{xyz}\,dxdydz =$

$\frac{\ln x}{yz}\Big|_1^{e^9} \Rightarrow \frac{\ln e^9}{y9} - \frac{\ln e}{yz}$

$\frac{9}{yz} - \frac{1}{yz}$ $\quad \frac{9}{yz} - \frac{1}{yz}$ $\quad 9\frac{\ln y}{z} - \frac{\ln y}{z}$

$\frac{8}{yz}$ $\quad \frac{8\ln y}{z}\Big|_1^{e^8}$ $\quad \frac{56}{z}$ $\quad \ln z$ $\quad 8\frac{\ln y}{z}$

(a) 288

(b) 432

(c) 48

(d) 144

8. If $\int_0^a \int_0^{\frac{x}{a}} \int_0^y dzdydx = 3$, then $a =$

$\int_0^a \int_0^{x/a} y.dy$

$\int_0^a \frac{y^2}{2}\Big|_0^{x/a}$

$\frac{x^2}{2a^2}\Big|_0^a$

$\frac{56}{z}$ $\quad 64 - 8$ $\quad 46 \ln z$ $\quad 46 \ln$

$2\times 63 - 63$ $\quad \frac{64}{z} - \frac{8}{z} \Rightarrow \frac{63}{z}$ $\quad 63 \ln z$ $\quad (63)2 - 63$ $\quad 63$

(a) 1

(b) 0

(c) 18

(d) 9

9. The area of the closed region bounded by the curve $r = 3 + 2\sin\theta$, $0 \le \theta \le 2\pi$ is

$\frac{x^3}{6a^2}\Big|_0^a \Rightarrow \frac{a^3}{6a^2}$

$\Rightarrow \frac{a}{6} = 3$

$a = 6 \times 3$

$a = 18$

(a) 22π

(b) 11π

(c) 9π

(d) 3π

$\int_0^{2\pi} \int_0^{3+2\sin\theta} r.drd\theta$

$\frac{1}{2}(3+2\sin\theta)^2$

$9\pi + 2\pi - 6 + 6$
$= 11\pi$

$\frac{r^2}{2}\Big|$

$\frac{1}{2}(9 + 12\sin\theta + 4\sin\theta$

$\frac{1}{2}\left(9\theta - 12\cos\theta + 2\theta - \frac{2\sin 2\theta}{2}\right)^{2\pi}$ $\quad \frac{1}{2}(9 + 12\sin\theta + 2 - 2\cos 2\theta)$

$\int_{-2} \int_{-\sqrt{4-x^2}} \int_0 r\,dz\,dr\,d\theta = \int_0 \int_0 (4r - r^3)\,dr\,d\theta = \int_{2\theta} \left[2r^2 - \frac{r^4}{4}\right]_0$

10. The volume of the solid bounded by the surfaces $z = 0$ and $z = 4 - x^2 - y^2$ is $\int_0 4\,d\theta = 8\pi$

   (a) $\frac{8}{3}\pi$

   (b) $\frac{32}{3}\pi$

   (c) $\frac{64}{3}\pi$

   (d) 2π

8π

$\begin{vmatrix} i & j & k \\ 1 & 2 & -2 \\ 5 & -2 & -1 \end{vmatrix}$

$i(-2-4) - j(-1+10) + k(-2-10)$

$-2i - 3j - 4k \quad -6i - 9j - 12k$

$\int_{x=-2}^{x=2} \int_{y=-\sqrt{4-x^2}}^{y=\sqrt{4-x^2}} \int_{z=0}^{z=4-r^2}$

$\int_0^2 \int_0^{2\pi} \int_0^{4-r^2} r\,dz\,d$

$r(4-r^2)$

$(4r - r^2)_2$

$2\pi\left(2(4) - \frac{8}{3}\right) \quad \left(\frac{4r^2}{2} - \frac{r^3}{3}\right) 2\pi$

$3 \times \frac{8}{3} - \frac{8}{3} \quad \left(2r^2 - \frac{r^3}{3}\right) 2\pi$

$\frac{32\pi}{3} \quad 2\left(\frac{16}{3}\right) \quad 24 - \frac{8}{3}$

11. The line of intersection of the planes $x + 2y - 2z = 5$, $5x - 2y - z = 0$

   (a) is perpendicular to the vector $\vec{u} = 2i + 4k$

   (b) is parallel to the vector $\vec{u} = 2i + 4k$

   (c) is parallel to the x-axis

   (d) the planes do not intersect

12. The line $L : x = 1 + 2t, y = 1 + 5t, z = 3t$ meets the plane $x + y + z = 2$ at the point

   (a) $(1, 1, 0)$

   (b) $(3, -1, 0)$

   (c) $(\frac{3}{2}, \frac{-3}{2}, 2)$

   (d) $(2, -1, 1)$

$v_1 = i + 2j - 2k$

$v_2 = 5 - 2j - k$

$1 + 2t + 1 + 5t + 3t = 2$

$2 +$

$t = 0$

$(1, 1, 0)$

$\begin{vmatrix} i & j & k \\ 1 & 2 & -2 \\ 5 & -2 & -1 \end{vmatrix}$

$x + 2y - 2z = 5$

$\vec{v_1} = i + 2j - k$

$\vec{v_2} = i$

$\nabla f =$

$i(-2-2) - j(-1+10) + k(-2-10)$

$-4i - 9j - 12k$

13. If $e^x \ln(x + y) = 1$, then $\frac{\partial y}{\partial x}$ at $(0, e) =$

   (a) e

   (b) $-e - 1$

   (c) -1

   (d) 1

$\frac{e^x \ln(x+y) + \frac{e^x}{x+y}}{\frac{e^x}{x+y}}$

$1 + \frac{1}{e}$

$\frac{dy}{dx} = -\frac{f_x}{f_y}$

$= \frac{-\left(\frac{e^x}{x+y} + e^x \ln(x+y)\right)}{\frac{e^x \cdot 1}{x+y}}$

$(0, e)$

$\frac{-\left(\frac{1}{e} + 1\right)}{\frac{1}{e}}$

14. The line $L : x = 1 + 2t, y = 1 + 5t, z = 3t$ meets the plane $x + y + z = 2$ at the point

   (a) $(1, 1, 0)$.

   (b) $(3, -1, 0)$.

   (c) $(\frac{3}{2}, \frac{-3}{2}, 2)$.

   (d) $(2, -1, 1)$

$1 + 2t + 1 + 5t + 3t = 2$

$2 + 5t = 2$

$t = 0$

$x = 1$

$y = 1$

$z = 0$

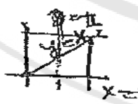
$(0, e)$

$\left(-\frac{1}{e} - 1\right) e \quad \frac{-\frac{1}{e} - 1}{\frac{1}{e}}$

$-\frac{e}{e} - e$

$-1 - e$

15. $\int_0^{\frac{\pi}{2}} \int_y^{\frac{\pi}{2}} \frac{\sin x}{x}\,dx\,dy =$



   (a) -1

   (b) 1

   (c) $1 - \cos 1$

   (d) $\sin 1$

$\int_{x=0}^{x=\frac{\pi}{2}} \int_{y=x}^{y=\frac{\pi}{2}} \frac{\sin x}{x}$

$\frac{-\left(e^y \ln(x+y) + \frac{e^x}{x+y}\right)}{\frac{e^x}{x+y}}$

$\frac{-\left(1 + \frac{1}{e}\right)}{\frac{1}{e}}$

16. The vector $v = ai + bj$ is parallel to the line $bx - ay = -c$. This statement is

   (a) True

   (b) False

$ab - ba$

$\int_0^{\frac{\pi}{2}} \frac{y \sin x}{x} \Big|_{y=x}^{y=\frac{\pi}{2}}$

$1 \quad -\sin x$

$-e - 1 = \frac{-\frac{e}{e} - \frac{1}{e}}{\frac{1}{e}}$

$bi - aj$

$d = mi + j = \frac{b}{a} x + \frac{c}{a}$

$$e^{x}\ln(x+y)=1 \qquad \frac{\partial y}{\partial x}$$

$$= -\frac{f_x}{f_y}$$

$$= \frac{-\left(e^{x}\ln(x+y)+\frac{e^{x}}{x+y}\right)}{\frac{e^{x}}{x+y}} \qquad (0,e)$$

$$= \frac{-\left(1\times\ln e+\frac{1}{e}\right)}{\frac{1}{e}}$$

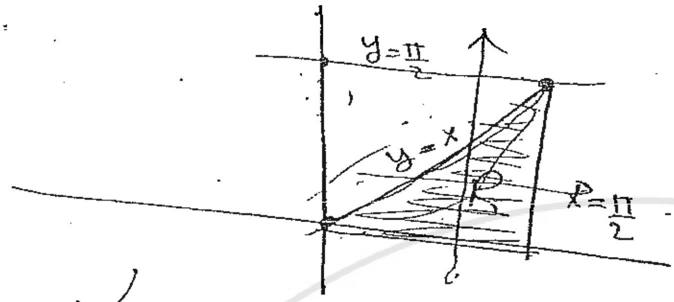
$$= -\frac{\left(1+\frac{1}{e}\right)}{\frac{1}{e}}$$

$$-e-1$$

$0 < y < y$

$0 < y < \frac{\pi}{2}$

$y < x < \frac{\pi}{2}$



$(\cos e)$

$(\cos e)$

$y=\frac{\pi}{2}$

$\int$

$y=x$

$e^x \ln(x+y) - 1 =$

$- \dfrac{\left(e^x \ln(x+y) + \frac{e^x}{x+y} \right)}{\frac{e^x}{x+y}}$

$\int_0^{\frac{\pi}{2}} \int_0^{x} \frac{\sin x}{x} \, dy\, dx$

$- \dfrac{\left(1 + \frac{1}{e}\right)}{\frac{1}{e}}$

$\sin x \Big|_0^{\frac{\pi}{2}}$

$\dfrac{-1 - \frac{1}{e}}{\frac{1}{e}}$

$\cos$

$\sin x = \cos x.$

$- \cos x \Big|_0^{\frac{\pi}{2}}$

$-0 + 1$

$-e - 1$

$\int_1^{e^2} \int_1^{e^8} \int_1^{e^9} \frac{1}{xyz} \, dz\, dy\, dx.$

$\ln z \Big|^{e^9}$

$\frac{9}{xy} - \frac{1}{xy} = \int_1^{e^8} \frac{8}{xy} \Rightarrow 8 \frac{\ln y}{x}$

$\frac{14}{8} - \frac{\overset{5}{64}}{\frac{8}{56}}$

$\frac{14}{\frac{8}{56}}$

$\frac{8 \times 8}{x} - \frac{8}{x}$

$\Rightarrow \frac{64}{x} - \frac{8}{x}$

$\frac{56}{x} \, dx \quad 56 \ln x.$

$56 \times 2 - 56$

$\Rightarrow 56$

17. If u, v, w are vectors in $\mathbb{R}^3$, then $(u \times v).w = (v \times w).u$. This statement is

(a) True

(b) False

$\begin{vmatrix} i & j & k \\ 3 & 4 & 2 \\ 1 & 2 & 0 \end{vmatrix}$ $i(0-4) - j(-1) + k(6-4)$

$-4i + 2j + 2k$

18. Let u, v be nonzero vectors, then a vector that is orthogonal to both $u+v$ and $u-v$ is

(a) $u \times (u+v)$ $= 0$

(b) $v \times (u+v)$ $= 0$

(c) $u \times v$ $=$ عمودي على الي

(d) none

$(u+v) \times (u-v)$

$\vec{u} = 3i + 4j + 2k$ $\quad u = 2i + 3j + k$

$\vec{v} = i + 2j$ $\quad v = i + j + k$

$-12 + 8 + 4$

19. The area of the triangle whose vertices are $A(-1,-1)$, $B(3,3)$, $C(2,1)$ is

(a) 1

(b) 2

(c) 3

(d) 4

$AB = 4i + 4j$

$AC = 3i + 2j$

$\begin{vmatrix} i & j & k \\ 4 & 4 & 0 \\ 3 & 2 & 0 \end{vmatrix}$ $i(0-0) - j(0) + k(8-12)$

$|4| = \frac{4}{2}$

20. An equation of the line that passes through $(1,1,1)$ and parallel to the z-axis is

parallel to z-axis

(a) $x = 1, y = 1, z = 1$

(b) $x = 1, y = 1, z = 1+t$

(c) $x = 1+t, y = 1, z = 1$

(d) $x = 1, y = 1+t, z = 1$

$(0$ $\quad x = 1+t \quad (0,0,1)$

$y = 1+t \quad (1,1,1)$

$= k \quad z = 1+t \quad z = 1 \quad = i + j$

$x = 1$

$y = 1$

21. An equation of the plane through $(2,4,5)$ perpendicular to the line $x - 5 = \frac{y-1}{3} = \frac{z}{4}$ is

(a) $3x + y + 4z = 34$

(b) $x + y + 12z = 34$

(c) $3x + 4y + z = 34$

(d) $x + 3y + 4z = 34$

$x - 5 = t$

$x = 2 + t$ $\quad x = 5 + t$ — ①

$y = 4 + 3t$ $\quad 3t + 1 = y$

$0 = (x-2) + 3(y-4) + 4(z-5)$ $\quad 4t = z$

$x - 2 + 3y - 12 + 4z - 20 = 0$

$x + 3y - 14 + 4z - 20$

$x + 3y + 4z = 34$

22. The point of intersection of the lines $L_1 : X = t, y = 3-3t, z = -2-t$ and $L_2 : X = 1+s, y = 4+s, z = -1+s$ is

(a) $(0,-3,2)$

(b) $(0,3,-2)$

(c) $(3,-2,0)$

(d) The lines do not intersect.

$z = -2$

$t = 1 + s$ $\quad s = -1$

$3 - 3t = 4 + s$ $\quad z = -2$

$-3 + 4t = -3$ $\quad t = 0$

$+3 \quad +3$

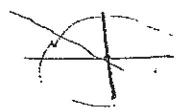
$(0, 3, -2)$

23. The point $(2, \frac{3\pi}{4})$ lies on the curve $r = 2\sin\theta$. This statement is

(a) True

(b) False

$2(\sin \frac{3\pi}{4})$



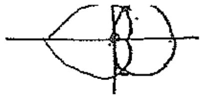
$2 \times \frac{1}{\sqrt{2}}$ $\frac{2}{\sqrt{2}}$

$\frac{\pi}{4}$

$\frac{270}{2}$ $\quad \frac{180}{4} \times$

$\leftarrow 135$ $\quad 3 \times 45 \quad 90$

$\leftarrow 135$



24. The curves $r = \cos\theta$ and $r = 1 - \cos\theta$ intersect in

(a) 1 point
(b) 2 points
(c) 3 points
(d) 4 points

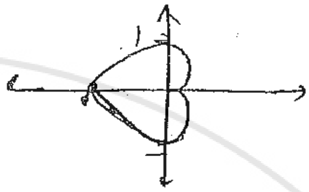
$1 - \cos\theta = \cos\theta$
$1 = 2\cos\theta$
$\cos\theta = \frac{1}{2}$

$\theta =$

25. The curves $r = 1 - \cos\theta$ and $r = -1 - \cos\theta$ have the same graph. This statement is

(a) False
(b) True

$\theta = \frac{\pi}{2}$
$\theta = \pi$



26. The length of the curve $r = \frac{1}{\sqrt{2}}e^{\theta}$, $0 \le \theta \le \pi$ is

(a) $1 - e^{\pi}$
(b) $e^{\pi} - 1$
(c) e^{π}
(d) 1

$\int \sqrt{\frac{1}{2}(e^{\theta})^2 + \frac{1}{2}(e^{\theta})^2}$

$\int_0^{\pi} e^{\theta}$

$\sqrt{(e^{\theta})^2} = 1 \quad | e^{\theta} | = e^{\pi} - 1$

27. The area that lies inside the circle $r = -2\cos\theta$ and outside the circle $r = 1$ is

(a) $\frac{\pi}{2} + 1$
(b) $\frac{3\pi}{2}$
(c) $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$
(d) $\frac{\pi}{3}$

$\cos\theta = -\frac{1}{2}$

$\frac{3\pi}{3} - \frac{\pi}{3} = \frac{2\pi}{3} \quad \theta = \frac{\pi}{3}$

$\int_{\frac{2\pi}{3}}^{\pi} (4\cos\theta^2) - 1$

$2 + 2\cos 2\theta - 1$

$1 + 2\cos 2\theta$

$\theta + \frac{2\sin 2\theta}{2}$

$\frac{2\pi}{3} + \sin(\frac{2\pi}{3}) - $
$(\pi + 0) - (\frac{2\pi}{3} - $

6 **Question 2** (6%). Find an equation for (a) the tangent plane and (b) for the normal line of the surface $x^2 + y^2 - 2xy - x + 3y - z = -4$ at the point $(2, -3, 18)$.

$f_x = 2x - 2y - 1 = 4 + 6 - 1 = 9.$
$f_y = 2y - 2x + 3 = -6 - 4 + 3 \Rightarrow -7.$
$f_z = -1$

$\nabla f = 9i - 7j - k.$ 2

tangent plane

$9(x-2) + -7(y+3) + -1(z-18) = 0$
$9x - \cancel{18} - 7y - 21 - z + \cancel{18} = 0$
$9x - 7y - z = 21.$ 2

normal tangent line

$x = 2 + 9t$
$y = -3 - 7t.$
$z = 18 - t.$ 2

$\int_{\frac{2\pi}{3}}^{\pi} (4\cos\theta^2 - 1) \cdot d\theta$

$\int_{\frac{2\pi}{3}}^{\pi} (2 + 2\cos 2\theta - 1) \cdot d\theta$

$\theta + \frac{2\sin 2\theta}{2}$

$\theta + \sin 2\theta \Big|_{\frac{2\pi}{3}}^{\pi}$

$\pi - [\frac{2\pi}{3} + $

$f(x,y) = e^{2x} \cos y$

$f_x = 2e^{2x} \cos y = 0$

$f_y = -e^{2x} \sin y = 0$

for x $(-1, -\infty)$

$y = \frac{n\pi}{2}$ critical point

$f_{xx} = 4e^{2x} \cos y = \begin{cases} 0, & n \text{ odd} \\ 0, & n \text{ even} \end{cases}$

$f_{yy} = -e^{2x} \sin y = 0$

$f_{xy} = -2e^{2x} \sin y = 0.$

$d = 0$

Question 3 (15%). Find all local maxima, local minima and saddle points of the function $f(x,y) = e^2x\cos y$.

$\overset{2x}{e}\cos y$.

8

$f(x,y) = e^{2x}\cos y$.

$f_x = 2e^{2x}\cos y = 0$ $\quad x \xrightarrow{\infty} -\infty \quad x \to (0, -\infty)$.

$f_y = -\sin y\, e^{2x} = 0$ $\quad y = n\pi$

$f_{xx} = 4e^{2x}\cos y = 0$

$f_{yy} = -\cos y\, e^{2x} = \begin{cases} e^{2x} = n \to \text{odd} \\ -e^{2x} = n \to \text{even} \end{cases} \to 0$

$f_{xy} = -2e^{2x}\sin y$

$d = f_{xx}f_{yy} - (f_{xy})^2$

$d = 0 - (0) = 0$ No information about Local max and Local min and Saddel point

---

~~$f_x = 2e^{2x}\cos y = 0$ $\quad y = \frac{n\pi}{2}$ $\quad x_1$~~

~~$f_y = 2e^{2x}\sin y$ $\quad y = n\pi$ $\quad x_2$~~

~~point~~

حل آخر

$2\cancel{e^{2x}}\cos y = -\sin y\,\cancel{e^{2x}}$

$-\tan y = 2$

$\tan y = -2$

$y = -63$

$f(x,y) = e^{2x}\cos y$ $\quad f_x = 2e^{2x}\cos y = 0 \Rightarrow \cos y = 0 \to (1)$

$f_y = -e^{2x}\sin y = 0 \Rightarrow \sin y = 0 \to (2)$

no pt. satisfy (1) & (2) so n critical pt.

حل سؤال ٦

$x^2+y^2+z^2=25$

$f(x,y,z)=x+2y+3z$

$1=2x\lambda$ — ①

$\lambda=\frac{1}{2x}$

$2=2y\lambda$

$\lambda=\frac{1}{y}$ — ②

$\frac{1}{2x}=\frac{1}{y} \Rightarrow y=2x.$

$3=2z\lambda$

$\lambda=\frac{3}{2z}$

$\frac{1}{\cancel{2}x}=\frac{3}{\cancel{2}z}$

$3x=z$

$x^2+4x^2+9x^2=25$

$14x^2=25$

$x=\pm\sqrt{\frac{25}{14}}$

$x=\pm\frac{5}{\sqrt{14}}$

$y=\pm\frac{10}{\sqrt{14}}$

$z=\pm\frac{15}{\sqrt{14}}$

$\frac{25}{14}+\frac{100}{14}+\frac{225}{14}$

$\Rightarrow$

point : $\left(\frac{5}{\sqrt{14}}, +\frac{10}{\sqrt{14}}, \frac{15}{\sqrt{14}}\right)$ maxsuim

$\left(-\frac{5}{\sqrt{14}}, \frac{-10}{\sqrt{14}}, \frac{15}{\sqrt{14}}\right)$ minumum

$f(x,y,z)=\frac{5}{\sqrt{14}}+\frac{20}{\sqrt{14}}+\frac{45}{\sqrt{14}}$

$=\frac{70}{\sqrt{14}}$ maxu at $\left(\frac{5}{\sqrt{14}}, \frac{10}{\sqrt{14}}, \frac{15}{\sqrt{14}}\right)$

$f(x_2,y_2,z_2)=-\frac{70}{\sqrt{14}}$

minumum

$\left(\frac{1}{2\lambda}\right)^2+\left(\frac{1}{\lambda}\right)^2+\left(\frac{3}{2\lambda}\right)^2=25$

$\frac{1}{4\lambda^2}+\frac{1}{\lambda^2}+\frac{9}{4\lambda^2}=25$

$\frac{14}{4\lambda^2}=25$

$\lambda^2=\frac{100}{14}$

$\boxed{\lambda=\mp\frac{10}{\sqrt{14}}}$

$x=\mp\frac{1}{2\lambda}=\frac{\sqrt{14}}{5}$

Question 4 (15%). Find the points on the sphere $x^2+y^2+z^2=25$ where the function $f(x,y,z)=x+2y+3z$ has its maximum and minimum values.

15

$g(x,y,z) = x^2+y^2+z^2-25$

$f(x,y) = x+2y+3z$

$1 = 2x\lambda \Rightarrow \lambda = \frac{1}{2x}$

$2 = 2y\lambda \Rightarrow \lambda = \frac{1}{y}$

$3 = 2z\lambda$

$\lambda = \frac{3}{2z}$

$\frac{1}{2x} = \frac{1}{y}$

$\boxed{y = 2x}$

$\frac{3}{2z} = \frac{1}{2x}$

$x = 3z$

$\frac{y}{2} = 3z$

$\boxed{y = 6z}$

$9z^2 + 36z^2 + z^2 = 25$

$46z^2 = 25$

خلف

في الصفحة الأخرى

1

Math 231

لاراد

Birzeit University
Mathematics Department
Final Test

Math 231 Calculus III
Dec 15, 2009

First Semester 2009/2010

(KEY)

Student Name: ____________________
Student Number: ____________________

Instructor: (Check only one box)
□ Abdul-Hamid Aburrub □ Aalaa Armiti □ Shadi Omari

19 × 3

57

Question # 1 (57 %)
Circle the letter that corresponds to the best answer for each question:

Use the information in the box to answer questions 1 – 3

Suppose that $f(x,y)$ is a differentiable function satisfying $$f(1,3)=1,\quad f_x(1,3)=2,\quad f_y(1,3)=4,$$

(1) Find a vector that is perpendicular to the level curve $f(x,y)=1$ at the point $(1,3)$.

(a) $2\hat{i}+2\hat{j}$ (b) $2\hat{i}-\hat{j}$ (c) $-\hat{i}+4\hat{j}$ (d) $\hat{i}+2\hat{j}$

(2) Find the derivative of the function $z=f(x,y)$ at the point $(1,3)$ in the direction of $-\hat{i}+2\hat{j}$.

(a) $\frac{3}{\sqrt{5}}$ (b) $\frac{6}{\sqrt{5}}$ (c) $\sqrt{5}$ (d) $2\sqrt{5}$

(3) Use linear approximation to estimate the value of $f(1.2,3.1)$.

(a) 2.1 (b) 0.4 (c) 1.8 (d) 1.4

(4) Find the volume of the solid region W, in the first octant, bounded from above by the plane $z=x+y$, and from the sides by the cylinder: $x^2+y^2=4$, and from below by the xy-plane.

(a) $\frac{16}{3}$ (b) 8 (c) 16 (d) $\frac{4}{3}$

Use the information in the box to answer questions 5 – 6

Suppose that the integral of a function over a region R is given in polar coordinates by

$\int_0^3 \int_0^{\pi/2} r^2 d\theta\, dr$

(5) Convert the integral to Cartesian coordinates

(a) $\int_0^3 \int_0^{\sqrt{9-x^2}} \sqrt{x^2+y^2}\, dy\, dx$ (b) $\int_0^3 \int_0^{\sqrt{9-x^2}} \left(x^2+y^2\right) dy\, dx$

(c) $\int_0^3 \int_0^3 \sqrt{x^2+y^2}\, dx\, dy$ (d) $\int_0^3 \int_0^{\sqrt{9-x^2}} \left(x^2+y^2\right) dx\, dy$

(6) Evaluate the integral, (in polar or in Cartesian).

(a) $\left(\frac{\pi}{2}\right)^3$ (b) 9π (c) $\frac{9\pi}{2}$ (d) 27π

Consider the ellipsoid $x^2+y^2+2z^2=4$ and the point $P(1,1,1)$

to answer questions 7 – 8

(7) Find parametric equations for the line that is normal to the ellipsoid at P.

(a) $x=1+t,\ y=1+t;\ z=1+2t$ (b) $x=1+2t,\ y=1+2t,\ z=1+2t$

(c) $x=2+t,\ y=2+2t,\ z=2+4t$ (d) $x=1+2t,\ y=1+t,\ z=1+4t$

(8) The line in question 7 intersects the ellipsoid in another point. Find that point.

(a) $\left(\frac{11}{5},\frac{11}{5},\frac{17}{5}\right)$ (b) $\left(\frac{11}{5},\frac{11}{5},\frac{-7}{5}\right)$ (c) $\left(\frac{-1}{5},\frac{-1}{5},\frac{17}{5}\right)$ (d) $\left(\frac{-1}{5},\frac{-1}{5},\frac{-7}{5}\right)$

(9) Compute the integral by reversing the order of integration

$$\int_0^1 \int_{\sqrt{y}}^1 \frac{y\, e^{x^2}}{x^3}\, dx\, dy$$

(a) $e-1$ (b) $\frac{e-1}{4}$ (c) $\frac{e^2-1}{4}$ (d) $\frac{e-1}{2}$

(10) If $f(x,y,z)=\left[z^2+z\sin(x-3y)\right]^2$, then compute $\frac{\partial f}{\partial x}$ at the point $\left(0,\frac{\pi}{3},1\right)$

(a) -1 (b) 1 (c) 2 (d) -2

(11) If $f(x,y,z)=x^2+2y-yz$, $\vec{u}=2\hat{i}+2\hat{j}-\hat{k}$ and $P(1,-2,1)$, then find $(D_{\vec{u}}f)_P$

(a) $\frac{13}{3}$ (b) $\frac{4}{3}$ (c) $\frac{11}{3}$ (d) $-\frac{1}{3}$

(12) Evaluate the integral in cylindrical coordinates

$$\int_0^{2\pi} \int_0^1 \int_0^{\sqrt{1-r^2}} zr\, dz\, dr\, d\theta$$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{8}$ (d) $\frac{\pi}{4}$

(13) Find the limit

$$\lim_{(x,y)\to(0,0)}\left[\frac{2xy}{x^2+2y^2}\right]$$

(a) 0 (b) 1 (c) ∞ (d) does not exist

Use the information in the box to answer questions 14 – 16

The position vector of a particle moving in the space is given by $\bar{r}(t)=(2\cos t)\,\hat{i}+(\sin t)\,\hat{j}+t\,\hat{k}$

(14) Find the velocity at $t=0$

(a) $\hat{i}+\hat{j}$ (b) $\hat{j}+\hat{k}$ (c) $2\hat{i}$ (d) $-2\hat{i}$

(15) Find the acceleration at $t=0$

(a) $\hat{i}+\hat{j}$ (b) $\hat{j}+\hat{k}$ (c) $2\hat{i}$ (d) $-2\hat{i}$

(16) Find the curvature at $t=0$

(a) 2 (b) $\sqrt{2}$ (c) 1 (d) $\frac{1}{2}\sqrt{2}$

(17) Find the volume, (use spherical coordinates), of the solid described by
$D=\{(x,y,z)\,|\,x^2+y^2+z^2\le 4, x\le 0, y\ge 0, z\le 0\}$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{8\pi}{3}$ (d) $\frac{4\pi}{3}$

(18) Find a local maximum of the function $f(x,y)=y$ on the curve $x^2+xy+y^2=3$

(a) 0 (b) 1 (c) 2 (d) -1

(19) If $w=\dfrac{1}{2x+y}$, and $x=t$, $y=-t$, find $\dfrac{dw}{dt}$ at the point $(1,-1)$.

(a) -1 (b) 1 (c) 2 (d) -2

Question # 2 14%

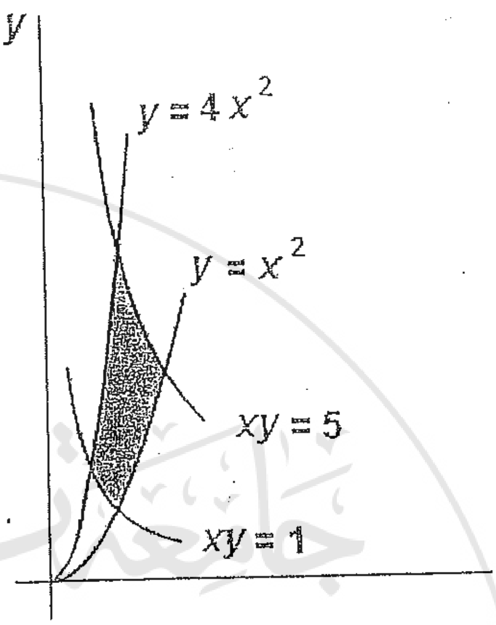
Evaluate the integral $\iint_R xy\, dA$ over the region bounded by the curves

$xy = 1$, $xy = 5$, $y = x^2$, and $y = 4x^2$

as in the figure

Use the transformation

$u = xy$, $v = x^2$



The Transformation

2 $u = xy \;,\; v = x^2$

The Inverse $\Rightarrow$ u

2 $x = \sqrt{v} \;,\; y = \dfrac{u}{\sqrt{v}}$

The Jacobian

2 $J = \begin{vmatrix} 0 & \frac{1}{2} v^{-\frac{1}{2}} \\ v^{-\frac{1}{2}} & -u\,\frac{1}{2} v^{-\frac{3}{2}} \end{vmatrix} = -\frac{1}{2} v^{-1} = \frac{-1}{2v}$

The region in the uv-plane

4 $xy = 1 \Longrightarrow u = 1 \;,\; xy = 5 \Longrightarrow u = 5$

$y = x^2 \Longrightarrow \dfrac{u}{\sqrt{v}} = v \Longrightarrow u = v^{3/2} \Rightarrow v = u^{2/3}$ $\quad \frac{1}{\sqrt[3]{u^2}} \quad \frac{1}{\sqrt[3]{16}}$

$\Longrightarrow u = 5v^{3/2} \Rightarrow v = \left(\frac{u}{4}\right)^{2/3}$

$y = 5x^2$

The integral

3 $\displaystyle\int_1^5 \int_{u^{2/3}}^{5v^{3/2}\;\left(\frac{u}{4}\right)^{2/3}} \sqrt{v}\cdot\frac{u}{\sqrt{v}}\cdot\frac{1}{2v}\, du\, dv$ $\quad dv\, du \cdot$ $\displaystyle\int_1^5\int_{\left(\frac{u}{4}\right)^{1/3}}^{u^{2/3}} \frac{u}{2v}\, dv\, du$

Evaluation

1 $\displaystyle\int \frac{1}{2v}\left[\frac{u^2}{2}\right]_{v^{3/2}}^{5v^{3/2}} dv$

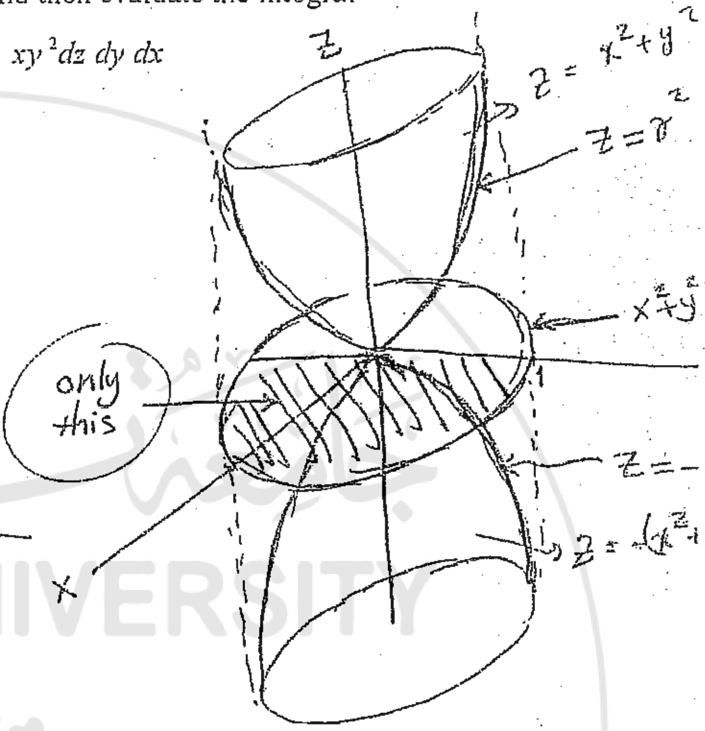
$\displaystyle = \int_1^5 6v^2\, dv = 248$

14

Question # 3 (14%)

Change the integral to cylindrical coordinate, and then evaluate the integral

$$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-(x^2+y^2)}^{(x^2+y^2)} xy^2\,dz\,dy\,dx$$



The region

$-r^2 \le z \le r^2$

$-\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$

$0 \le x \le 1$

The integral

8 $\int \int \int x y^2 \, dz\, dy\, dx$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^1 \int_{-r^2}^{r^2} r\cos\theta\,(r\sin\theta)^2\, dz\; r\,dr\,d\theta$$

Evaluation

6 $\int \int \int_{-r^2}^{r^2} r^4 \cos\theta \sin^2\theta \, dz\, dr\, d\theta$

$= \int \int_0^1 2r^6 \cos\theta \sin^2\theta \, dr\, d\theta$ $\quad \frac{r^7}{7}$

let $u = \sin$ $\;\; du = \cos$

$= \int \frac{2}{7} \cos\theta \sin^2\theta \, d\theta$ $\quad 2$

$= \frac{2}{7} \left.\frac{\sin^3\theta}{3}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$ $= \frac{4}{21}$

14

Question # 4 (16%)

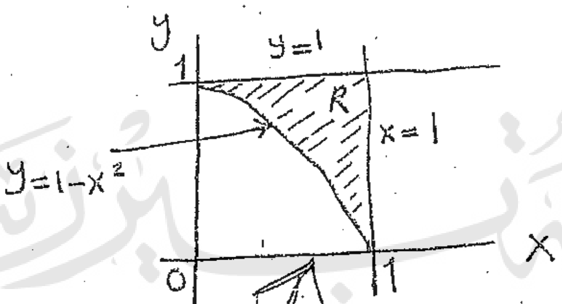
Find the average value of the function $f(x, y) = 2xy$ over the region R enclosed by the curves and lines: $y = 1 - x^2$, $y = 1$, and $x = 1$

2

$$av = \text{Average Value} = \frac{1}{\text{Area}(R)} \iint_R f(x,y)\,dA$$

The Region R

4



4 $$\text{Area}(R) = \int_0^1 \int_{1-x^2}^1 \boxed{dy\,dx}$$

2 $$\text{Value of the area} = \int [1-(1-x^2)]\,dx$$
$$= \int_0^1 x^2\,dx = \frac{1}{3}$$

The integral

3 $$\iint f(x,y)\,dA = \int_0^1 \int_{1-x^2}^1 2xy\,dy\,dx$$

$$= \int x\left[1-(1-x^2)^2\right]dx$$

$$= \int_0^1 (2x^3 - x^5)\,dx$$

$$= \frac{2}{4} - \frac{1}{6}$$

$$= \frac{1}{3}$$

1 $av =$ $\qquad \frac{1}{1/3} \cdot \frac{1}{3} = 1$

16

Birzeit University
Mathematics Department
Final Test

Math 231 Calculus III
Dec 15, 2009

First Semester 2009/2010

(KEY)

Student Name: ______________________
Student Number: ______________________

Instructor: (Check only one box)
☐ Abdul-Hamid Aburrub ☐ Aalaa Armiti ☐ Shadi Omari

19*3
57

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(2) Find the derivative of the function $z=f(x,y)$ at the point $(1,3)$ in the direction of $-\hat{i}+2\hat{j}$

(a) $\frac{3}{\sqrt{5}}$ (b) $\frac{6}{\sqrt{5}}$ (c) $\sqrt{5}$ (d) $2\sqrt{5}$

(3) Use linear approximation to estimate the value of $f(1.2,3.1)$.

(a) 2.1 (b) 0.4 (c) 1.8 (d) 1.4

(4) Find the volume of the solid region W, in the first octant, bounded from above by the plane $z=x+y$, and from the sides by the cylinder $x^2+y^2=4$, and from below by the xy-plane.

(a) $\frac{16}{3}$ (b) 8π (c) 16π (d) $\frac{4\pi}{3}$

Use the information in the box to answer questions 5 – 6

Suppose that the integral of a function over a region R is given in polar coordinates by $\int_0^3 \int_0^{\pi/2} r^2 d\theta\, dr$

(5) Convert the integral to Cartesian coordinates

(a) $\int_0^3 \int_0^{\sqrt{9-x^2}} \sqrt{x^2+y^2}\, dy\, dx$ (b) $\int_0^3 \int_0^{\sqrt{9-x^2}} (x^2+y^2)\, dy\, dx$

(c) $\int_0^3 \int_0^3 \sqrt{x^2+y^2}\, dx\, dy$ (d) $\int_0^3 \int_0^{\sqrt{9-x^2}} (x^2+y^2)\, dx\, dy$

(6) Evaluate the integral, (in polar or in Cartesian).

(a) $\left(\frac{\pi}{2}\right)^3$ (b) 9π (c) $\frac{9\pi}{2}$ (d) 27π

Consider the ellipsoid $x^2+y^2+2z^2=4$ and the point $P(1,1,1)$ to answer questions 7 – 8

(7) Find parametric equations for the line that is normal to the ellipsoid at P.

(a) $x=1+t,\ y=1+t,\ z=1+2t$ (b) $x=1+2t,\ y=1+2t,\ z=1+2t$

(c) $x=2+t,\ y=2+2t,\ z=2+4t$ (d) $x=1+2t,\ y=1+t,\ z=1+4t$

(8) The line in question 7 intersects the ellipsoid in another point. Find that point.

(a) $\left(\frac{11}{5},\frac{11}{5},\frac{17}{5}\right)$ (b) $\left(\frac{11}{5},\frac{11}{5},\frac{-7}{5}\right)$ (c) $\left(\frac{-1}{5},\frac{-1}{5},\frac{17}{5}\right)$ (d) $\left(\frac{-1}{5},\frac{-1}{5},\frac{-7}{5}\right)$

(9) Compute the integral by reversing the order of integration

$$\int_0^1 \int_{\sqrt{y}}^1 \frac{y\, e^{x^2}}{x^3}\, dx\, dy$$

(a) $e-1$ (b) $\frac{e-1}{4}$ (c) $\frac{e^2-1}{4}$ (d) $\frac{e-1}{2}$

(10) If $f(x,y,z)=\left[z^2+z\sin(x-3y)\right]^2$, then compute $\frac{\partial f}{\partial x}$ at the point $\left(0,\frac{\pi}{3},1\right)$

(a) -1 (b) 1 (c) 2 (d) -2

(11) If $f(x,y,z)=x^2+2y-yz$, $\bar{u}=2\hat{i}+2\hat{j}-\hat{k}$ and $P(1,-2,1)$, then find $(D_{\bar{u}}f)_P$

(a) $\frac{13}{3}$ (b) $\frac{4}{3}$ (c) $\frac{11}{3}$ (d) $-\frac{1}{3}$

(12) Evaluate the integral in cylindrical coordinates

$$\int_0^{2\pi}\int_0^1\int_0^{\sqrt{1-r^2}} zr\, dz\, dr\, d\theta$$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{8}$ (d) $\frac{\pi}{4}$

(13) Find the limit

$$\lim_{(x,y)\to(0,0)}\left[\frac{2xy}{x^2+2y^2}\right]$$

(a) 0 (b) 1 (c) ∞ (d) does not exist

Use the information in the box to answer questions 14 – 16

The position vector of a particle moving in the space is given by $\vec{r}(t) = (2\cos t)\hat{\mathrm{i}} + (\sin t)\hat{\mathrm{j}} + t\,\hat{\mathrm{k}}$

(14) Find the velocity at $t = 0$

(a) $\hat{\mathrm{i}} + \hat{\mathrm{j}}$ (b) $\hat{\mathrm{j}} + \hat{\mathrm{k}}$ (c) $2\hat{\mathrm{i}}$ (d) $-2\hat{\mathrm{i}}$

(15) Find the acceleration at $t = 0$

(a) $\hat{\mathrm{i}} + \hat{\mathrm{j}}$ (b) $\hat{\mathrm{j}} + \hat{\mathrm{k}}$ (c) $2\hat{\mathrm{i}}$ (d) $-2\hat{\mathrm{i}}$

(16) Find the curvature at $t = 0$

(a) 2 (b) $\sqrt{2}$ (c) 1 (d) $\frac{1}{2}\sqrt{2}$

(17) Find the volume, (use spherical coordinates), of the solid described by
$D = \{(x, y, z) \mid x^2 + y^2 + z^2 \le 4, x \le 0, y \ge 0, z \le 0\}$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{8\pi}{3}$ (d) $\frac{4\pi}{3}$

(18) Find a local maximum of the function $f(x, y) = y$ on the curve $x^2 + xy + y^2 = 3$

(a) 0 (b) 1 (c) 2 (d) -1

(19) If $w = \dfrac{1}{2x + y}$, and $x = t$, $y = -t$, find $\dfrac{dw}{dt}$ at the point $(1, -1)$.

(a) -1 (b) 1 (c) 2 (d) -2

Question # 2 14%

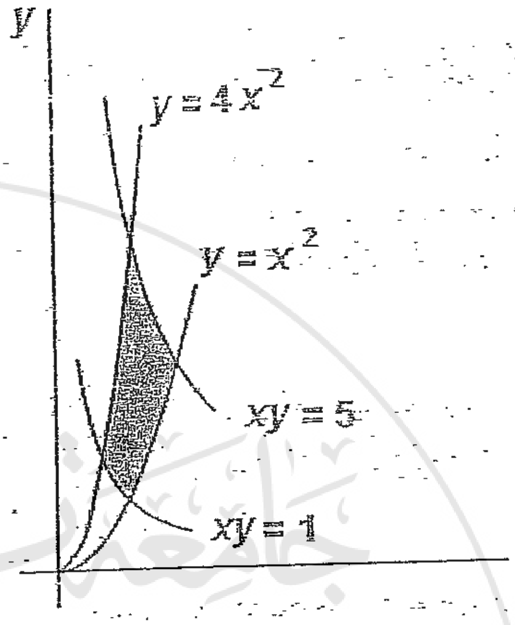
Evaluate the integral $\iint_R xy\,dA$ over the region bounded by the curves

$xy = 1$, $xy = 5$, $y = x^2$, and $y = 4x^2$

as in the figure

Use the transformation

$u = xy$, $v = x^2$



The Transformation

2 $\quad u = xy \,,\; v = x^2$ $\quad u$

The Inverse

2 $\quad x = \sqrt{v} \,,\; y = \frac{u}{\sqrt{v}}$

The Jacobian

2 $\quad J = \begin{vmatrix} 0 & \frac{1}{2} v^{-\frac{1}{2}} \\ v^{-\frac{1}{2}} & -u \frac{1}{2} v^{-\frac{3}{2}} \end{vmatrix} = -\frac{1}{2} v^{-1} = \frac{-1}{2}$

The region in the uv-plane

4 $\quad xy = 1 \Longrightarrow u = 1 \,,\quad xy = 5 \Longrightarrow u = 5$

$y = x^2 \Longrightarrow \frac{u}{\sqrt{v}} = v \Longrightarrow u = v^{3/2} \Rightarrow v = u^{2/3}$

$y = 5x^2 \Longrightarrow u = 5 v^{3/2} \Rightarrow v = \left(\frac{u}{4}\right)^{2/3}$

The integral

3 $\quad \int_1^5 \int_{u^{2/3}}^{(u/4)^{2/3}} \sqrt{v} \cdot \frac{u}{\sqrt{v}} \cdot \frac{1}{2v}\, \cancel{du\,dv}\; dv\,du \cdot \int_1^5 \int_{(u/4)^{2/3}}^{u^{2/3}} \frac{u}{2v}\, dv\, du$

Evaluation

1 $\quad \int \frac{1}{2v} \left[\frac{u^2}{2}\right]_{v^{3/2}}^{5v^{3/2}} dv$

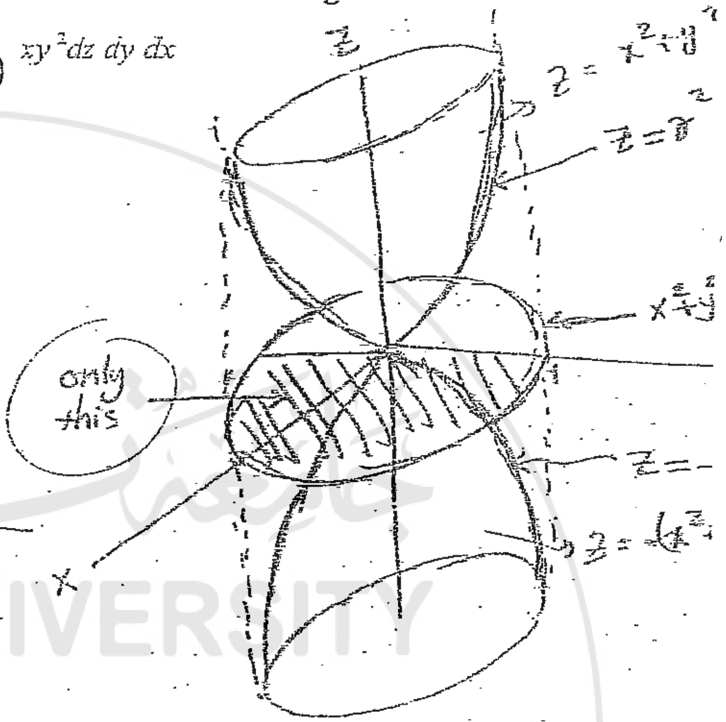
$= \int_1^5 6 v^2\, dv = 248$

14

Question #3 (14%)

Change the integral to cylindrical coordinate, and then evaluate the integral

$$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-(x^2+y^2)}^{(x^2+y^2)} xy^2\,dz\,dy\,dx$$



The region

$-r^2 \le z \le r^2$

$-\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$

$0 \le x \le 1$

The integral

8

$$\int\int\int xy^2\,dz\,dy\,dx$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0 \int_{-r^2}^{r^2} r\cos\theta\,(r\sin\theta)^2\,dz\,r\,dr\,d\theta$$

Evaluation

6

$$\int\int\int_{-r^2}^{r^2} r^4\cos\theta\sin^2\theta\,dz\,dr\,d\theta$$

$$= \int\int_0^1 2r^6\cos\theta\sin^2\theta\,dr\,d\theta \qquad \frac{r^7}{7}$$

let $u = \sin$ $du = \cos$

$$= \int \frac{2}{7}\cos\theta\sin^2\theta\,d\theta$$

$$= \frac{2}{7}\left[\frac{\sin^3\theta}{3}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{4}{21}$$

14

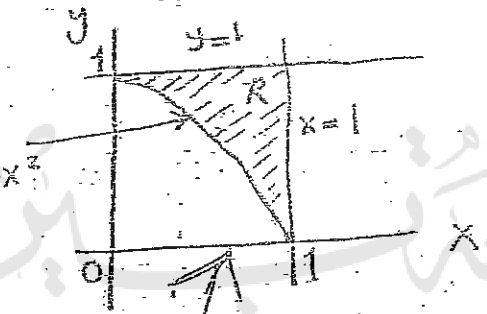
Question # 4 (16%)

Find the average value of the function $f(x,y) = 2xy$ over the region R enclosed by the curves and lines: $y = 1 - x^2$, $y = 1$, and $x = 1$

2

$$av = \text{Average Value} = \frac{1}{\text{Area}(R)} \iint_R f(x,y)\,dA$$

The Region R



4

$$\text{Area}(R) = \int_0^1 \int_{1-x^2}^{1} \boxed{dy\,dx}$$

4

$$\text{Value of the area} = \int [1-(1-x^2)]\,dx$$

$$= \int_0^1 x^2\,dx = \frac{1}{3}$$

2

The integral

$$\iint f(x,y)\,dA = \int_0^1 \int_{1-x^2}^{1} 2xy\,dy\,dx$$

3

$$= \int x\left[1-(1-x^2)^2\right]dx$$

$$= \int_0^1 (2x^3 - x^5)\,dx$$

$$= \frac{2}{4} - \frac{1}{6}$$

$$= \left(\frac{1}{3}\right)$$

1 $\quad av =$ $\qquad \dfrac{1}{1/3} \cdot \dfrac{1}{3} = 1$

16

$24(2) = 48 + 6 + 8 + 15 + 10 = 85$

Birzeit University
Mathematics Department
Math 231 - Final Exam
Instructors: Dr. Khaled Al-Takhman, Mr. Rasim Kaabi

Summer 2006

Student Name: ....Asmaa Nael Aror.................. Number: 1050853 Section: ....

Dr Rasem Kaabi

Question 1 (54%). Circle the most correct answer:

1. If a region R in the plane is symmetric about the x-axis and the y-axis, then

   (a) $\int\int_R f(x,y)\ ;dxdy = 2\int\int_G f(x,y)\ dxdy$, G is the part of R in the first quadrant.

   (b) $\int\int_R f(x,y)\ ;dxdy = 4\int\int_G f(x,y)\ dxdy$, G is the part of R in the first quadrant.

   (c) $\int\int_R f(x,y)\ ;dxdy = 4\int\int_G f(x,y)\ dxdy$, G is the part of R in the first and second quadrants.

2. An equation of the line that passes through the point $(-8,-3)$ perpendicular to $\vec{v} = -5i + 4j$ is

   $-5(x+8) + 4(y+3) = 0$

   $-5x - 40 + 4y + 12 = 0$

   $-5x + 4y = 28$

   (a) $-5x + 4y = 28$.

   (b) $y + 3 = \frac{-4}{5}(x+8)$.

   (c) $-4x + 5y = 17$.

   (d) $-5x + 4y = 41$.

   $t - 6 = s$

   $t = s$

3. The lines $L_1 : x = t - 6, y = t, z = 2t$ and $L_2 : x = t, y = t, z = -t$

   $\langle 1, 1, 2\rangle$

   $\langle 1, 1, -1\rangle$

   not parallel

   $t_1 - 6 = t_2$

   $t_1 = t_2$

   $0 - 6 =$

   (a) intersect at a point

   (b) are parallel

   (c) not parallel and do not have intersection point

   (d) none.

   $1 - 6$

   $t - 6 = t_1$

   $t = t_1$

   $0 - 6 = 0$

4. When reversing the order of integration of $\int_0^4\int_0^{\frac{5y}{4}} dxdy$ we get

   $x = \frac{5}{4}y$

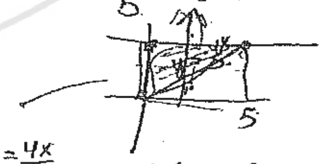
   $\frac{4x}{5} = y$, $0 < y < 4$

   (a) $\int_0^5\int_0^{\frac{5x}{4}} dydx$

   (b) $\int_0^5\int_0^{\frac{x}{4}} dydx$

   (c) $\int_0^5\int_{\frac{4x}{5}}^4 dydx$

   (d) $\int_0^{\frac{5y}{4}}\int_0^4 dydx$



$x =$

$y = 0$

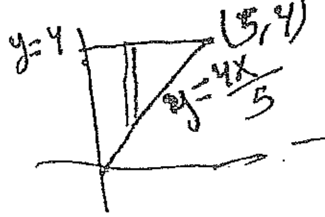
$y = 4$

$\int_0^5\int_{y=0}^{y=\frac{4x}{5}} dy\,dx$

$4 = \frac{4}{5}x$

$x = 5$

$\int_{x=0}^{x=5}\int_{y=\frac{4x}{5}}^{y=4}$



$$\int_1^{2}\int_1^{8}\int_1^{e^9} \frac{1}{xyz}\,dx\,dy\,dz$$

$$\frac{\ln x}{yz}\Big|_1^{e^9}$$

$$\Rightarrow \frac{9}{yz} - \frac{1}{yz} \Rightarrow \frac{8}{yz} \Rightarrow \frac{8\ln y}{z}$$

$$\frac{8(8)}{z} - \frac{8}{z}$$

$$\begin{array}{r} 64 - \\ 18 \\ \hline 46 \end{array}$$

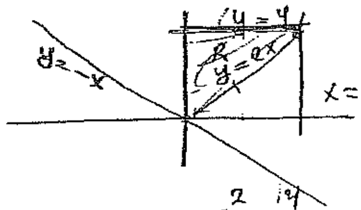
$$\frac{64-8}{z} \Rightarrow$$

$$\frac{46}{z} \Rightarrow 46\ln z$$

$$46 \times 2 - 46$$

$$\Rightarrow 46$$

5. $\int_0^4\int_{\frac{y}{2}}^{2} f(x,y)\,dxdy + \int_{-2}^{0}\int_{-y}^{2} f(x,y)\,dxdy =$



$2x = y$

(a) $\int_0^2\int_{-x}^{2x} f(x,y)\,dydx$

$\int_0^2\int_{2x}^{4} f(x,y)\cdot dydx$

(b) $\int_0^4\int_{-2x}^{x} f(x,y)\,dydx$

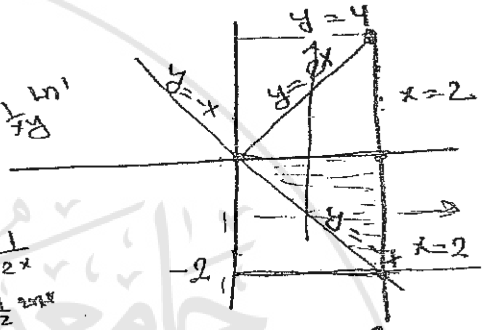
$x = 2$
$y = -x$

(c) $\int_0^2\int_{-x}^{x} f(x,y)\,dydx$

$x =$

$x = 2$
$2x =$

(d) $\int_0^2\int_{-2}^{4} f(x,y)\,dydx$



6. $\int_{-7}^{7}\int_{-\sqrt{49-y^2}}^{\sqrt{49-y^2}} dxdy =$ قطبية

$\frac{9\ln y}{x}$ $\frac{1}{xy}$ $\frac{9\times 8}{x}$

(a) 7π

$\int_0^{2\pi}\int_0^{7} r\cdot dr d\theta$

(b) 49π

$\int_0^{2\pi} \frac{r^2}{2} d\theta \Big|_0^7$

(c) 196π

(d) 98π

$\frac{49}{2}\theta \Big|_0^{2\pi}$ $\frac{49}{2}(2\pi)$

49π

$\int_{y=-x}^{y=2x}\int_0^2 dxdy$

7. $\int_1^{e^2}\int_1^{e^8}\int_1^{e^9} \frac{1}{xyz}\,dxdydz =$

(a) 288

$\frac{\ln x}{yz}\Big|_1^{e^9} \Rightarrow \frac{\ln e^9}{yz} - \frac{\ln e}{yz}$

(b) 432

(c) 48

$\frac{9}{yz} - \frac{1}{yz}$ $\frac{56}{z}$ $\frac{9}{yz} - \frac{1}{yz}$ $\frac{9\ln y}{z} - \frac{\ln y}{z}$

(d) 144

$\frac{8}{yz}$ $\frac{8\ln y}{z}\Big|_1^{e^8}$ $\ln z$ $8\frac{\ln y}{z}$

8. If $\int_0^a\int_0^{\frac{x}{a}}\int_0^y dzdydx = 3$, then $a =$

$\frac{56}{z}$ $64-8$

(a) 1

$\int_0^a\int_0^{x/a} y\cdot dy$ $56\ln z$

(b) 0

$\int_0^a \frac{y^2}{2}\Big|_0^{x/a}$ $\frac{64}{z} - \frac{8}{z} \Rightarrow \frac{63}{z}$

(c) 18

$2\times 63 - 63$ $63\ln z$

(d) 9

$\frac{x^2}{2a^2}\Big|^a$ $(63)2 - 63$

9. The area of the closed region bounded by the curve $r = 3 + 2\sin\theta$, $0 \le \theta \le 2\pi$ is

63

(a) 22π

$\frac{x^3}{6a^2}\Big|_0^a \Rightarrow \frac{a^3}{6a^2}$

(b) 11π

$\Rightarrow \frac{a}{6} = 3$

(c) 9π

(d) 3π

$a = 6\times 3$

$a = 18$

$9\pi + 2\pi - 6 + 6 = 11\pi$

$\int_0^{2\pi}\int_0^{3+2\sin\theta} r\cdot dr d\theta$

$\frac{1}{2}(3+2\sin\theta)^2$

$\frac{r^2}{2}\Big|$ $\frac{1}{2}(9 + 12\sin\theta + 4\sin\theta)$

$\frac{1}{2}\left(9\theta - 12\cos\theta + 2\theta - \frac{2\sin 2\theta}{2}\right)\Big|^{2\pi}$ $\frac{1}{2}(9 + 12\sin\theta + 2 - 2\cos 2\theta)$

10. The volume of the solid bounded by the surfaces $z = 0$ and $z = 4 - x^2 - y^2$ is

(a) $\frac{8}{3}\pi$

(b) $\frac{32}{3}\pi$

(c) $\frac{64}{3}\pi$

(d) 2π

11. The line of intersection of the planes $x + 2y - 2z = 5$, $5x - 2y - z = 0$

(a) is perpendicular to the vector $\vec{u} = 2i + 4k$

(b) is parallel to the vector $\vec{u} = 2i + 4k$

(c) is parallel to the x-axis

(d) the planes do not intersect

12. The line $L : x = 1 + 2t, y = 1 + 5t, z = 3t$ meets the plane $x + y + z = 2$ at the point

(a) $(1, 1, 0)$

(b) $(3, -1, 0)$

(c) $(\frac{3}{2}, \frac{-3}{2}, 2)$

(d) $(2, -1, 1)$

13. If $e^x \ln(x + y) = 1$, then $\frac{\partial y}{\partial x}$ at $(0, e) =$

(a) e

(b) $-e - 1$

(c) -1

(d) 1

$\frac{dy}{dx} = -\frac{F_x}{F_y}$

14. The line $L : x = 1 + 2t, y = 1 + 5t, z = 3t$ meets the plane $x + y + z = 2$ at the point

(a) $(1, 1, 0)$.

(b) $(3, -1, 0)$.

(c) $(\frac{3}{2}, \frac{-3}{2}, 2)$.

(d) $(2, -1, 1)$

$1+2t + 1+5t+3t = 2$

$t = 0$

$x = 1$, $y = 1$, $z = 0$

15. $\int_0^{\frac{\pi}{2}} \int_y^{\frac{\pi}{2}} \frac{\sin x}{x}\, dx dy =$

(a) -1

(b) 1

(c) $1 - \cos 1$

(d) $\sin 1$

16. The vector $v = ai + bj$ is parallel to the line $bx - ay = -c$. This statement is

(a) True

(b) False

$ab - ba$

$$e^{x} \ln(x+y) = 1 \qquad \frac{\partial y}{\partial x}$$

$$= -\frac{f_x}{f_y}$$

$$= -\frac{\left(e^{x} \ln(x+y) + \frac{e^{x}}{x+y}\right)}{\frac{e^{x}}{x+y}} \qquad (0,e)$$

$$= -\frac{\left(1 \times \ln e + \frac{1}{e}\right)}{\frac{1}{e}}$$

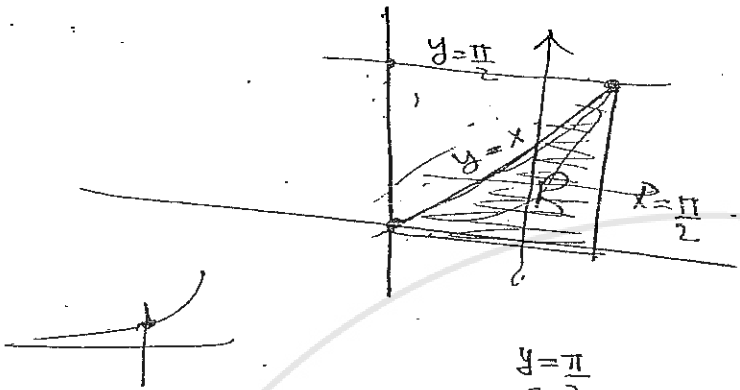
$$= -\frac{\left(1 + \frac{1}{e}\right)}{\frac{1}{e}}$$

$$-e-1$$

$0 < y < y$

$0 < y < \frac{\pi}{2}$

$y < x < \frac{\pi}{2}$



(cos e)

(cos e)

$$\int_{y=x}^{y=\frac{\pi}{2}}$$

$$e^{x} \ln(x+y) - 1 =$$

$$\frac{-\left(e^{x}\ln(x+y) + \frac{e^{x}}{x+y}\right)}{\frac{e^{x}}{x+y}}$$

$$\int_0^{\frac{\pi}{2}} \int_0^{x} \frac{\sin x}{x} \, dy\,dx$$

$$\frac{-\left(1 + \frac{1}{e}\right)}{\frac{1}{e}}$$

$$\sin x \Big|_0^{\frac{\pi}{2}}$$

$$\frac{-1 - \frac{1}{e}}{\frac{1}{e}}$$

cos

$\sin x = \cos x$.

$$-\cos x \Big|_0^{\frac{\pi}{2}}$$

$-0 + 1$

$-e - 1$

$$\int_1^{e^2} \int_1^{e^8} \int_1^{e^9} \frac{1}{xyz} \, dz\,dy\,dx$$

$$\ln z \Big|^{e^9}$$

$$\frac{9}{xy} - \frac{1}{xy} = \int_1^{e^8} \frac{8}{xy} \Rightarrow \frac{8 \ln y}{x}$$

$\frac{14}{8} - \frac{\cancel{64}^{5}}{\frac{8}{56}} -$

$\frac{14}{8} - \frac{}{56}$

$$\frac{8 \times 8}{x} - \frac{8}{x}$$

$$\Rightarrow \frac{64}{x} - \frac{8}{x}$$

$$\frac{56}{x} \, dx \quad 56 \ln x.$$

$$56 \times 2 - 56$$

$$\Rightarrow 56$$

17. If u, v, w are vectors in $\mathbb{R}^3$, then $(u \times v).w = (v \times w).u$. This statement is

(a) True

(b) False

$i(0-4) - j(-2) + k(6-4)$ $\begin{vmatrix} i & j & k \\ 3 & 4 & 2 \\ 1 & 2 & 0 \end{vmatrix}$

$-4i + 2j + 2k$

18. Let u, v be nonzero vectors, then a vector that is orthogonal to both $u+v$ and $u-v$ is

(a) $u \times (u+v)$ $= 0$

(b) $v \times (u+v)$ $= 0$

(c) $u \times v$ $=$

(d) none

$(u+v) \times (u-v)$

$\vec{u} = 3i + 4j + 2k$ $u = 2i + 3j + k$

$\vec{v} = i + 2j$ $v = i + j + k$

$-12 + 8 + 4$

19. The area of the triangle whose vertices are $A(-1,-1), B(3,3), C(2,1)$ is

(a) 1

(b) 2

(c) 3

(d) 4

$AB = 4i + 4j$

$AC = 3i + 2j$

$\begin{vmatrix} i & j & k \\ 4 & 4 & 0 \\ 3 & 2 & 0 \end{vmatrix}$ $i(0-0) - j(0) + k(8-12)$

$|4| = \frac{4}{2}$

20. An equation of the line that passes through $(1,1,1)$ and parallel to the z-axis is

parallel to z-axis

(a) $x = 1, y = 1, z = 1$

(b) $x = 1, y = 1, z = 1 + t$

(c) $x = 1 + t, y = 1, z = 1$

(d) $x = 1, y = 1 + t, z = 1$

$x = 1 + t$ $(0,0,1)$

$y = 1 + t$ $(1,1,1)$

$z = 1$ $= i + j$

k $z = 1 + t$

$x = 1$

$y = 1$

21. An equation of the plane through $(2,4,5)$ perpendicular to the line $x - 5 = \frac{y-1}{3} = \frac{z}{4}$ is

(a) $3x + y + 4z = 34$

(b) $x + y + 12z = 34$

(c) $3x + 4y + z = 34$

(d) $x + 3y + 4z = 34$

$x = 2 + t$ $x - 5 = t$

$y = 4 + 3t$ $x = 5 + t$ — ①

$0 = (x-2) + 3(y-4) + 4(z-5)$ $3t + 1 = y$

$x - 2 + 3y - 12 + 4z - 20 = 0$ $4t = z$

$x + 3y - 14 + 4z - 20$

22. The point of intersection of the lines $L_1 : X = t, y = 3 - 3t, z = -2 - t$ and $L_2 : X = 1 + s, y = 4 + s, z = -1 + s$ is

$x + 3y + 4z = 34$

(a) $(0, -3, 2)$

(b) $(0, 3, -2)$

(c) $(3, -2, 0)$

(d) The lines do not intersect.

$z = -2$

$t = 1 + s$

$3 - 3t = 4 + s$ $s = -1$ $z = -2$

$-3 + 4t = -3$ $t = 0$

$(0, 3, -2)$

23. The point $(2, \frac{3\pi}{4})$ lies on the curve $r = 2\sin\theta$. This statement is

(a) True

(b) False

$2\left(\sin\frac{3\pi}{4}\right)$

$2 \times \frac{1}{\sqrt{2}}$ $\frac{2}{\sqrt{2}}$

$\frac{\pi}{4}$

$\frac{180}{4}$

$\frac{270}{2}$

$\leftarrow 135$ 3×45 90

$\leftarrow 135$

24. The curves $r = \cos\theta$ and $r = 1 - \cos\theta$ intersect in

(a) 1 point
(b) 2 points
(c) 3 points
(d) 4 points

$1 - \cos\theta = \cos\theta$
$1 = 2\cos\theta$
$\cos\theta = \frac{1}{2}$

25. The curves $r = 1 - \cos\theta$ and $r = -1 - \cos\theta$ have the same graph. This statement is

(a) False
(b) True

$\theta = \frac{\pi}{2}$
$\theta = \pi$

26. The length of the curve $r = \frac{1}{\sqrt{2}}e^{\theta}$, $0 \le \theta \le \pi$ is

(a) $1 - e^{\pi}$
(b) $e^{\pi} - 1$
(c) e^{π}
(d) 1

$\int \sqrt{\frac{1}{2}(e^{\theta})^2 + \frac{1}{2}(e^{\theta})^2}$

$\sqrt{(e^{\theta})^2} = |e^{\theta}| = e^{\pi} - 1$

27. The area that lies inside the circle $r = -2\cos\theta$ and outside the circle $r = 1$ is

(a) $\frac{\pi}{2} + 1$
(b) $\frac{3\pi}{2}$
(c) $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$
(d) $\frac{\pi}{3}$

$\cos\theta = -\frac{1}{2}$
$\frac{3\pi}{3} - \frac{\pi}{3} = \frac{2\pi}{3}$ $\quad \theta = \frac{\pi}{3}$

$\int_{2\pi/3}^{\pi} (4\cos\theta^2) - 1$

$2 + 2\cos 2\theta - 1$
$1 + 2\cos 2\theta$
$\theta + \frac{2\sin 2\theta}{2}$

$\frac{2\pi}{3} + \sin(\frac{2\pi}{3}) - 1$
$(\pi + 0) - (\frac{2\pi}{3}$

6 **Question 2** (6%). Find an equation for (a) the tangent plane and (b) for the normal line of the surface $x^2 + y^2 - 2xy - x + 3y - z = -4$ at the point $(2, -3, 18)$.

$f_x = 2x - 2y - 1 = 4 + 6 - 1 = 9.$
$f_y = 2y - 2x + 3 = -6 - 4 + 3 \Rightarrow -7.$
$f_z = -1$

$\nabla f = 9i - 7j - k.$ 2

tangent plane

$9(x-2) + -7(y+3) + -1(z-18) = 0$
$9x - \cancel{18} - 7y - 21 - z + \cancel{18} = 0$
$9x - 7y - z = 21.$ 2

normal ~~tangent~~ line

$x = 2 + 9t$
$y = -3 - 7t.$
$z = 18 - t.$ 2

$\int_{2\pi/3}^{\pi} (4\cos\theta^2 - 1)\, d\theta$

$\int_{2\pi/3}^{\pi} (2 + 2\cos 2\theta - 1) \cdot d\theta$

$\theta + \frac{2\sin 2\theta}{2}$

$\theta + \sin 2\theta \Big|_{2\pi/3}^{\pi}$

$\pi - [\frac{2\pi}{3} +$

$$f(x,y) = e^{2x} \cos y$$

$$f_x = 2e^{2x}\cos y = 0$$

$$f_y = -\cancel{2}e^{2x}\sin y = 0$$

for $x \; (-1, -\infty)$

$y = \dfrac{n\pi}{2}$ critical point

$$f_{xx} = 4e^{2x}\cos y = \begin{cases} 0, & \cancel{n \neq 0} \;\; n \text{ odd} \\ 0, & n \text{ even} \end{cases}$$

$$f_{yy} = -e^{2x}\sin y = 0$$

$$f_{xy} = -2e^{2x}\sin y = 0.$$

$$d = 0$$

Question 3 (15%). Find all local maxima, local minima and saddle points of the function $f(x, y) = e^2x\cos y$. $e^{2x}\cos y$.

$x \to -\infty$

$f(x,y) = e^{2x}\cos y$.

$f_x = 2e^{2x}\cos y = 0$

$f_y = -\sin y\, e^{2x} = 0$

$x \xrightarrow{-\infty} -\infty$ $x \to (0, -\infty)$.

$y = n\pi$

$f_{xx} = 4e^{2x}\cos y = 0$

$f_{yy} = -\cos y\, e^{2x} = \begin{cases} e^{2x} = n \to \text{odd} \\ -e^{2x} = n \to \text{even} \end{cases} \to 0$

$f_{xy} = -2e^{2x}\sin y$

$d = f_{xx}f_{yy} - (f_{xy})^2$

$d = 0 - (0) = 0$ No information about local max and local min and saddel point

---

~~$f_x = 2e^{2x}\cos y = 0$ $y = \frac{n\pi}{2}$ x_1~~

~~$f_y = 2e^{2x}\sin y$ $y = n\pi$ x_2~~

~~point~~

حل آخر

$2\cancel{e^{2x}}\cos y = -\sin y\,\cancel{e^{2x}}$

$-\tan y = 2$

$\tan y = -2$

$y = -63$

$f(x,y) = e^{2x}\cos y$ $f_x = 2e^{2x}\cos y = 0 \Rightarrow \cos y = 0 \to ①$

$f_y = -e^{2x}\sin y = 0 \Rightarrow \sin y = 0 \to ②$

no pt. satisfy ① & ② so no critical pt.

حل سؤال (٤)

$x^2+y^2+z^2=25$

$f(x,y,z)=x+2y+3z$

$1=2x\lambda$ — (1)

$\lambda=\frac{1}{2x}$

$2=2y\lambda$

$\lambda=\frac{1}{y}$ — (2)

$\frac{1}{2x}=\frac{1}{y} \Rightarrow y=2x.$

$3=2z\lambda$

$\lambda=\frac{3}{2z}$

$\frac{1}{\cancel{2}x}=\frac{3}{\cancel{2}z}$

$3x=z$

$x^2+4x^2+9x^2=25$

$14x^2=25$

$x=\pm\sqrt{\frac{25}{14}}$

$x=\pm\frac{5}{\sqrt{14}}$

$y=\pm\frac{10}{\sqrt{14}}$

$z=\pm\frac{15}{\sqrt{14}}$

$\frac{25}{14}+\frac{100}{14}+\frac{225}{14}$

$\Rightarrow$

point : $\left(\frac{5}{\sqrt{14}}, \frac{+10}{\sqrt{14}}, \frac{15}{\sqrt{14}}\right)$ maxsum

$\left(-\frac{5}{\sqrt{14}}, \frac{-10}{\sqrt{14}}, \frac{15}{\sqrt{14}}\right)$ minumum

$f(x,y,z)=\frac{5}{\sqrt{14}}+\frac{20}{\sqrt{14}}+\frac{45}{\sqrt{14}}$

$=\frac{70}{\sqrt{14}}$ maxu at $\left(\frac{5}{\sqrt{14}}, \frac{10}{\sqrt{14}}, \frac{15}{\sqrt{14}}\right)$

$f(x_2,y_2,z_2)=-\frac{70}{\sqrt{14}}$

minumum

$\left(\frac{1}{2\lambda}\right)^2+\left(\frac{1}{\lambda}\right)^2+\left(\frac{3}{2\lambda}\right)^2=25$

$\frac{1}{4\lambda^2}+\frac{1}{\lambda^2}+\frac{9}{4\lambda^2}=25$

$\frac{14}{4\lambda^2}=25$

$\lambda^2=\frac{10}{14}$

$\boxed{\lambda=\mp\frac{10}{\sqrt{14}}}$

$x=\mp\frac{1}{2\lambda}=\frac{\sqrt{14}}{5}$

Question 4 (15%). Find the points on the sphere $x^2+y^2+z^2=25$ where the function $f(x,y,z) = x+2y+3z$ has its maximum and minimum values.

15

$g(x,y,z) = x^2+y^2+z^2-25$

$f(x,y) = x+2y+3z$

$1 = 2x\lambda \Rightarrow \lambda = \frac{1}{2x}$

$2 = 2y\lambda \Rightarrow \lambda = \frac{1}{y}$

$3 = 2z\lambda$

$\lambda = \frac{3}{2z}$

$\frac{1}{2x} = \frac{1}{y}$

$\boxed{y = 2x}$

$\frac{3}{2z} = \frac{1}{2x}$

$x = 3z$

$\frac{y}{2} = 3z$

$\boxed{y = 6z}$

$9z^2 + 36z^2 + z^2 = 25$

$46z^2 = 25$

ملاحظة خلف

في الصفحة الأخرى