13.1 Curves in Space and Their tangents

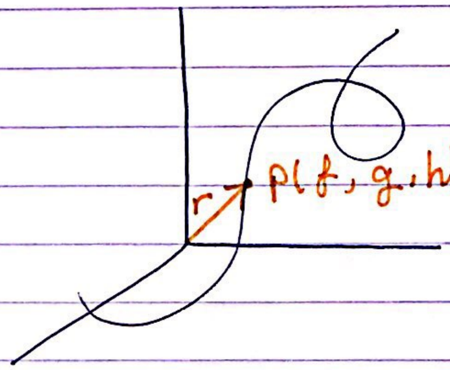
- A curve in the space can be represented in vector form:-

$$r(t) = \vec{OP} = f(t)\,\hat{\imath} + g(t)\,\hat{\jmath} + h(t)\,\hat{k}$$

$x = f(t)$

$y = g(t)$

$z = h(t)$



- limits and continuity

$$\lim_{t \to t_0} \vec{r}(t) = L_1\hat{\imath} + L_2\hat{\jmath} + L_3\hat{k}$$

$$\lim_{t \to t_0} f(t) = L_1$$

$\vec{r}(t)$ is cont if: $\lim_{t \to t_0} \vec{r}(t) = \vec{r}(t_0)$

$$\lim_{t \to t_0} g(t) = L_2$$

$$\lim_{t \to t_0} h(t) = L_3$$

- Derivatives.

if $\vec{r}'(t)$ is cont and $\neq 0$ Then curve $\vec{r}(t)$ is smooth

- Velocity and speed

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = |\vec{v}(t)| \frac{\vec{v}(t)}{|\vec{v}(t)|} \leftarrow \text{Direction}$$

speed

- To find parametric eqs of tangent line:-

Position(r) →

$$x = x_0 + v_1 t$$
$$y = y_0 + v_2 t$$
$$z = z_0 + v_3 t$$

Notes:

$$\ln(f(x))' = \frac{f(x)'}{f(x)}$$

$$\tan^{-1} x = \frac{1}{1+x^2}$$

13.2: Integrals of vector functions

$$\int r'(t)\, dt = \vec{R}(t) + \vec{C}$$

found by initial conditions

Some old integrals :-

$$\int \ln x \, dx = x \ln x - x$$

$$\int x e^x \, dx = x e^x - e^x$$

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x$$

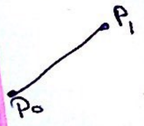
$$\int \frac{1}{1+x^2}\, dx = \tan^{-1} x$$

$$\int \sec x \, dx = \ln |\sec x - \tan x|$$

$$\int \sec^2 x \, dx = \tan x$$

- If you have two points and you want to find velocity of the particle moving.

$P_0 (P_1, P_2, P_3)$, $P_1 (P_1', P_2', P_3')$



$$\boxed{V = |(P_0 P_1)| \left(\frac{\overrightarrow{P_0 P_1}}{|P_0 P_1|} \right)}$$

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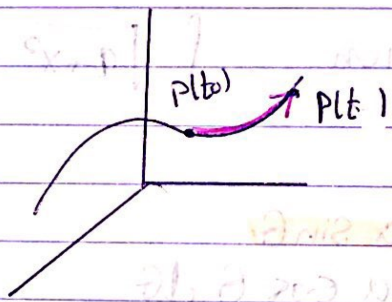
13.3 Arc length in Space

The length of the curve : $\vec{r}(t)$

$$L = \int_{t_0 \rightsquigarrow a}^{t_1 \rightsquigarrow b} \sqrt{(f'(t))^2 + (g'(t))^2 + (h'(t))^2}\, dt$$

Arc length parameter

$$S(t) = \int_{t_0}^{t} |v(\tau)|\, d\tau$$



- S = distance from $P(t_0)$ to $P(t)$ if $t > t_0$

- S = − distance if $t < t_0$

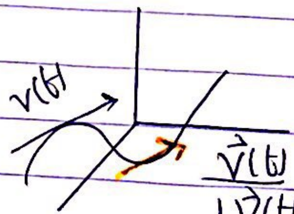
→ You can parametrize r by s : $r(t(s))$

- if $\vec{r}(t)$ is a smooth curve $\Rightarrow |\vec{v}| > 0$ and $S(t)$ has an inverse and in increasing

$$\frac{dt}{ds} = \frac{1}{|\vec{v}(t)|}$$

Unit tangent vector

$$\vec{T} = \frac{\vec{v}(t)}{|\vec{v}(t)|}$$



$$\frac{\vec{v}(t)}{|\vec{v}(t)|} = \vec{T}$$

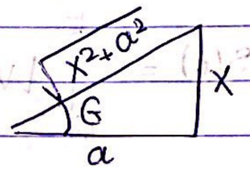
Remark: $\frac{d\vec{r}}{ds} = \frac{\vec{v}(t)}{|\vec{v}(t)|}$

• *Note:* Trigonometric substitutions

$x = a\tan\theta$

$dx = a\sec^2\theta \, d\theta$ $\qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

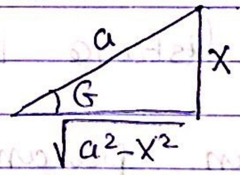
when: $\int \sqrt{a+x^2} \, dx$



$x = a\sin\theta$

$dx = a\cos\theta \, d\theta$

when: $\int \sqrt{a^2-x^2} \, dx$



$x = a\sec\theta$

$dx = a\sec\theta\tan\theta \, d\theta$

when $\int \sqrt{x^2-a^2} \, dx$