

13.4: Curvature and Normal vector

Curvature

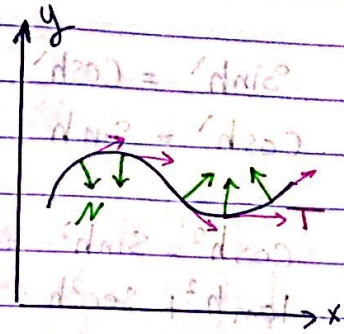
$$K = \frac{1}{|\vec{v}|} \left| \frac{d\vec{T}}{dt} \right|$$

→ for a circle:

$$K = \frac{1}{a}$$

→ if K is constant → circle curve \perp

→ $K=0$ if the curve is a line



Unit Normal vector:

$$N = \frac{dT/ds}{|dT/ds|} \rightarrow \text{perpendicular to } T$$

• N points in the direction where T turns

• T point to the concave side of the curve

But using chain rule:

$$N = \frac{dT/dt}{|dT/dt|}$$

Circle of curvature of plane curve:

$$\text{Radius of circle} = \text{Radius of curvature} = \rho = \frac{1}{K}$$

Revise Ex 1 in Book

$$\text{Circle Eq: } (x-x_0)^2 + (y-y_0)^2 = R^2$$

R : Radius
 (x_0, y_0) → Center

Revision

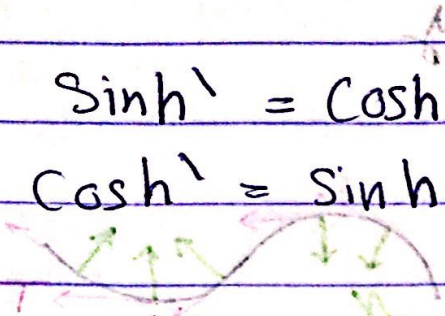
$$\sinh^{-1} = \cosh$$

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$$\cosh^2 - \sinh^2 = 1$$

$$\tanh^2 + \operatorname{sech}^2 = 1$$

$$\operatorname{cosech}^2 - \operatorname{csch}^2 = 1$$



$\frac{4}{b}$	$\frac{1}{\sqrt{1}}$
$\frac{1}{b}$	$\frac{1}{\sqrt{1}}$

If the curve is a line, the area under the curve is constant.

Normal vector

T of the curve is perpendicular to T

At point on the curve

where T is normal

T point to the curve

side of the curve

Chain rule

$\frac{dT}{ds}$

$\frac{dT}{ds}$

13.5: Tangential & Normal components of acceleration.

$$\vec{a} = \frac{d\vec{v}}{dt} = a_T \vec{T} + a_n \vec{N}$$

$$a_T = \frac{d|\vec{v}|}{dt}$$

$$a_n = \frac{|\vec{v}|^2}{R} = \frac{1}{|\vec{v}|} \left| \frac{d\vec{v}}{dt} \right| |\vec{v}|^2$$

$$\text{or } a_n = \sqrt{|\vec{a}|^2 - a_T^2}$$

Torsion: $T = \frac{\begin{vmatrix} x' & y' & z' \\ x'' & y'' & z'' \\ x''' & y''' & z''' \end{vmatrix}}{|\vec{v} \times \vec{a}|^2} = \frac{K}{|\vec{v}|^3}$

$$B = T \times N$$

