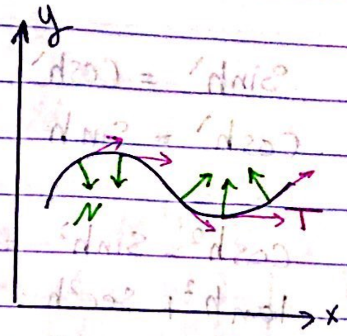
13.4: Curvature and Normal vector:



Curvature

$$K = \frac{1}{|\vec{v}|} \left| \frac{d\vec{T}}{dt} \right|$$

→ for a circle:

$$K = \frac{1}{a}$$

→ If K is constant → ال curve متجانس

→ $K = 0$ if the curve is a line

Unit Normal vector:

$$N = \frac{dT/ds}{|dT/ds|}$$

→ perpendicular to T

- N points in the direction where T turns
- T point to the concave side of the curve

But using chain rule:

$$N = \frac{dT/dt}{|dT/dt|}$$

Circle of curvature of a plane curve:

Radius of circle = Radius of curvature = $\rho = \frac{1}{K}$

Revise Ex 4 in Book

Circle Eq: $(x - x_0)^2 + (y - y_0)^2 = R^2$

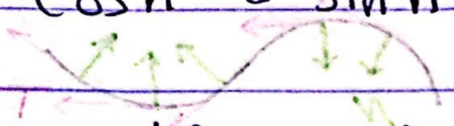
R: Radius

(x_0, y_0) → Center

Revision:

$\sinh' = \cosh$

$\cosh' = \sinh$



$\cosh^2 - \sinh^2 = 1$

$\tanh^2 + \operatorname{sech}^2 = 1$

$\coth^2 - \operatorname{csch}^2 = 1$

13.5: Tangential & Normal components of acceleration.

$$\vec{a} = \frac{d\vec{v}}{dt} = a_T \vec{T} + a_n \vec{N}$$

$$\boxed{a_T = \frac{d|\vec{V}|}{dt}} \qquad \boxed{a_n = K(|\vec{V}|)^2}$$

$$= \frac{1}{|V|}\left|\frac{dT}{dt}\right| \; |\vec{V}|^2$$

or $a_n = \sqrt{|a|^2 - a_T^2}$

Torsion : $\tau = \dfrac{\begin{vmatrix} x' & y' & z' \\ x'' & y'' & z'' \\ x''' & y''' & z''' \end{vmatrix}}{|V \times a|^2}$ $= K|V|^3$

$B = T \times N$