14.1 functions of several variables

- Treating The domain as a Region R:

Region

↓ **Closed**: If It contains all it's boundary points ⇒ محدود Boundaryال function ال حدود

↓ **open**: If It consists entirely of interior points

↓ **Bounded/unBounded**

Bounded:- If it lies iside a disk of a fixed radius

Ex: triangles / interiors of triangles, rectangles, circles and disks

UnBounded:-

Ex: lines, coordinate axes functions defined on infinite integrals plane, half planes and Quadrants

level curves and surfaces:-

level curve: The set of points in which:

$f(x,y) = C$ (C is constant)

level surface: The set of points in space (x,y,z) in which $f(x,y,z) = C$

surface ⇒ $z = f(x,y)$

14.2 limits and continuity in higher dimensions

Suppose: $\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y) = L$ $\quad$ $\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y) = M$

Then 1- $\lim\limits_{(x,y)\to(x_0,y_0)} \left(f(x,y) \pm g(x,y)\right) = L \pm M$

2- $\lim\limits_{(x,y)\to(x_0,y_0)} \left(f(x,y)\, g(x,y)\right) = LM$

3- $\lim\limits_{(x,y)\to(x_0,y_0)} \dfrac{f(x,y)}{g(x,y)} = \dfrac{L}{M}$, $M \neq 0$

4- $\lim\limits_{(x,y)\to(x_0,y_0)} c\, f(x,y) = cL$

5- $\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y)^n = L^n$

Solving limits using Polar Coordinates:

You put $x = r\cos\theta$ $\quad$ $y = r\sin\theta$

and so limit becomes: $\lim\limits_{r\to 0} f(r\cos\theta, r\sin\theta)$

Now if the answer is in terms of θ

Example $\lim\limits_{r\to 0} f(r\cos\theta, r\sin\theta) = \cos 2\theta$

- Then your limit is undefined because θ has infinite values

Solving limits by assuming $y = mx$

اذا الجواب كان بدلالة m بكون ال limit غير معرفة

Continuity :-

let $(x_0, y_0) \in D_f$, Then $f(x,y)$ is cont at (x_0, y_0) if :-

① $f(x_0, y_0)$ exists

② $\lim\limits_{(x_0, y_0)} f(x,y)$ exists

③ $\lim\limits_{(x,y) \to (x_0, y_0)} f(x,y) = f(x_0, y_0)$

Two path test for nonexistance of limits

if $f(x,y)$ has two different limits along two paths then the limit of f Does not exist

← يعني اذا ع لنا نختبار لل limit لطريقتين Polar و نفرض $y=mx$

Sandwich Theorem

$$h < f < g$$

$$\lim_{(x,y) \to (x_0, y_0)} h = L \qquad \lim_{(x,y) \to (x_0, y_0)} g = L$$

Then $\lim\limits_{(x,y) \to (x_0, y_0)} f = L$