14.3 Partial Derivatives

Def :

$$\left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)} = \lim \frac{f(x_0+h,\ y_0) - f(x_0,y_0)}{h}$$

partial derivative

$$\left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)} = \left.\frac{d\ f(x_0,y_0)}{dx}\right|_{x=x_0}$$

- $\frac{\partial f}{\partial x}$ معناها نشتق القانون بالنسبة لـ x ، ونعامل y كأنه constant
- اذا كان f حاصل ضرب اقترانين فيهم x مثل $x\sin xy$ نعمل مشتقة الضرب لانه x موجود في الجهتين

Implicit function.

لو عندنا function فيه x وy و z وبدنا $\frac{\partial z}{\partial x}$

مثل $yz - \ln z = x + y$

نشتق بالنسبة لـ x الطرفين حيث نعامل y انو ثابت ونشتق z بالنسبة لـ x و x بالنسبة لـ x

ف :

$$yz - \ln z = x + y \quad \text{كأنه constant}$$

$$y\frac{\partial z}{\partial x} - \frac{1}{z}\frac{\partial z}{\partial x} = 1 + 0$$

$$\frac{\partial z}{\partial x}\left(y - \frac{1}{z}\right) = 1$$

$$\frac{\partial z}{\partial x} = \frac{1}{y - \frac{1}{z}}$$

$\to f(x,y,z)$ And $f(x)$ or $f(y)$

• اذا بدنا نحسب ميل مماس لنقطة التقاء بين اقترانين
ميل مماس = مشتقة

← في هذه الحالة: مشتقة الاقتران بالنسبة لـ y أو x بعد تعويض $f(x)$ أو $f(y)$ محل x, y في الاقتران $f(x,y,z)$ هي ميل المماس

مثلاً:

$z = x^2 + y^2$ and plane $x = 1$

slope of tangent at $(1,2,5)$

١- نعوض x في z : $z = 1 + y^2$

٢- نشتق z بالنسبة لـ y ونعوض (2) من النقطة (1,2,5)

$\boxed{\frac{\partial z}{\partial y}}$ = slope $= 0 + 2y\Big|_{y=2} = 4$

Partial derivatives and continuity :-

• $f_x, f_y\big|_p$ (partial derivatives) can exist and still that won't mean that f_x is continuous at P.

مثلاً:

$$f(x,y) = \begin{cases} 0 & \Rightarrow xy \neq 0 \\ 1 & \Rightarrow xy = 0 \end{cases}$$

لو احسبنا limit عند (0,0) :

$\lim\limits_{(x,y)\to(0,0)} = 0$ and $f(0,0) = 1$

So f is not continous : ولكن

$$\frac{\partial f(0,0)}{\partial x} = \lim \frac{f(0+h,0) - f(0,0)}{h} = \frac{1-1}{h} = 0$$

$$\frac{\partial f(0,0)}{\partial y} = \lim \frac{f(0,h) - f(0,0)}{h} = 0$$

$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ exist

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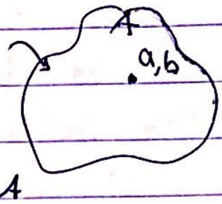
Mixed derivative Theorem :-

$$f_{xy}(a,b) = f_{yx}(a,b)$$

If f is continous at (a,b)

f_x, f_y
f_{xy}, f_{yz}
are defined
on Region A



Differentiability : $z = f(x,y)$

f is differentiable if $f_x(x_0,y_0)$, $f_y(x_0,y_0)$ exist and Δz satisfies equation:

$$\Delta z = f_x(x_0,y_0)\Delta x + f_y(x_0,y_0)\Delta y + \epsilon_1 \Delta x + \epsilon_2 \Delta y$$

in which $\epsilon_1, \epsilon_2 \to 0$ as $\Delta x, \Delta y \to 0$

and if f is differentiable at (x_0, y_0) then It's continous at (x_0, y_0)

14.4 : The chain rule

Chain rule for functions of <u>two</u> independant variables:-

$$w = f(x,y) \qquad x = x(t) \qquad y = y(t)$$

Then : $$\frac{dw}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

of three independant variables:

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

But if $w = f(x)$ and $x = g(r,s)$

Then

$$\frac{\partial w}{\partial r} = \frac{dw}{dx}\frac{\partial x}{\partial r}$$

$$\frac{\partial w}{\partial s} = \frac{dw}{dx}\frac{\partial x}{\partial s}$$

Implicit Differentiation:

$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$ where $F(x,y)=0$ defines y as a differentiable function of x $(F_y \neq 0)$

and so is

$$\underbrace{\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \qquad \frac{\partial z}{\partial y} = \frac{-F_y}{F_z}}_{}$$

here $F(x,y,z) = 0 \Rightarrow F(x,y,f(x,y)) = 0$

يعني مثلاً:- $x^3 + z^2 + ye^{xz} + z\cos y = 0$ وبدنا $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \Longrightarrow F_x = 3x^2 + 0 + zye^{xz} + 0 = 0$$

$F_y = 0 + 0 +$ [illegible] $e^{xz} + -z\sin y = 0$

$$\frac{\partial z}{\partial x} = -\frac{3x^2 + 2ye^{xz}}{e^{xz} - z\sin y}$$ بالتالي

و $\frac{\partial z}{\partial y}$ بنفس الطريقة

14.5 : Directional Derivatives and Gradient Vectors

The directional derivative $(D_u f)_{P_0}$

$$(D_u f)_{P_0} \left(\frac{df}{ds}\right)_{u,P_0} = \lim_{s \to 0} \frac{f(x_0 + s u_1, y_0 + s u_2) - f(x_0, y_0)}{s}$$

where the derivative of f at P_0 in the direction of vector $u = u_1 \hat{i} + u_2 \hat{j}$

$$(D_u f)_{P_0} = \vec{\nabla} f \cdot \vec{u} = \left(\frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j}\right) \cdot \left(u_1 \hat{i} + u_2 \hat{j}\right)$$

$$= |\nabla f| \underbrace{|u|}_{1} \cos\theta = |\nabla f| \cos\theta$$

Properties of the directional derivative :

1- function f increases most rapidly when $\theta = 0$ and so u is in the direction of ∇f

2- function f decreases most rapidly when $\theta = \pi$ and so u is in the direction opposite from ∇f

3- function f has zero change if $\theta = \frac{\pi}{2}$ and so u is orthogonal to ∇f

Note:

$\Rightarrow$ if V is a vector : $V = a\hat{i} + b\hat{j}$

n_1 (orthogonal on V) $= b\hat{i} - a\hat{j}$

n_2 (" " ") $= -b\hat{i} + a\hat{j}$

$\Rightarrow$ You can find direction of $u = \frac{\nabla f}{|\nabla f|}$ (if ∇f, u are on the same directi)

- If you have a function $f(x,y)$

→ ∇f is orthogonal to curve

→ and to find tangent to curve at point (x_0, y_0)

$$f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0) = 0$$

Note: Product rule : $\nabla(fg) = f\nabla g + g\nabla f$