14.6:- Tangent planes and differentials

Tangent plane :-

to function $f(x,y,z) = c$ and at Point $P_0(x_0, y_0, z_0)$
The equation of tangent plane is:-

$$f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0) + f_z(P_0)(z - z_0) = 0$$

مشتقة بالنسبة x

Normal line:-

The equation:

$$x = x_0 + f_x(P_0)t$$
$$y = y_0 + f_y(P_0)t$$
$$z = z_0 + f_z(P_0)t$$

• ∇f is orthogonal to the tangent plane

* If the function was $z = f(x,y)$, $P_0(x_0, y_0, f(x_0, y_0))$
then the equation of tangent line is:-

$$f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) - (z - z_0) = 0$$

Question idea :-

If there are two surfaces meeting in a specific shape E then the parametric equations for the line tangent at E at $P_0(x_0, y_0, z_0)$

find $V = \nabla f \times \nabla g$ (Direction of tangent plane)
and equations are then:-

$$x = x_0 + V_1 t, \quad y = y_0 + V_2 t, \quad z = z_0 + V_3 t$$

Estimating change in specefic direction:-

$$df = (\nabla f|_{P_0} \cdot u)\ ds$$

Where ; $u = \dfrac{\vec{PP_0}}{|PP_0|}$

∇f: Directional derivative

u : Direction of movement • رايح من النقطة

ds: distane increment التانية الى النقطة الأولى

linearization a function of two variables:-

function $f(x,y)$ at $P_0(x_0,y_0)$

$$L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

→ The error in the standard linear approximation

$$|E(x,y)| \leq \frac{1}{2} M \left(|x-x_0| + |y-y_0|\right)^2$$

where E is the error

M is the upperbound for the values $|f_{xx}|$, $|f_{yy}|$, $|f_{xy}|$

⇒ to find M, we put values of x', y' to make f_{xx}, f_{yy}, f_{xy} the highest possible value

x', y' is taken from Given $|x| \leq x'$, $|y| \leq y'$

Differentials

Total differentiation of f

$$df = f_x(x_0,y_0)\,dx + f_y(x_0,y_0)\,dy$$

where df is the resulting change of moving from (x_0,y_0) to (x_0+dx, y_0+dy)

Back to Example 9

14.7 Extreme values and Saddle point

Derivative tests for local extreme values

- local maximum: if $f(a,b) \geq f(x,y)$ for all domain points (x,y) in an open disk centered at (a,b)
- local minimum: 〃 〃 $\leq$ 〃 〃 〃
  〃 〃 〃

→ First derivative test for local extreme values
$f(a,b)$ is a local max or min if the first derivatives exist → $f_x(a,b)=0$, $f_y(a,b)=0$

Second derivative test for local extreme values

1- local max at (a,b) if $f_{xx}<0$ and $f_{xx}f_{yy}-f_{xy}^2>0$ at (a,b)

2- local min at (a,b) if $f_{xx}>0$ and $f_{xx}f_{yy}-f_{xy}^2>0$ at (a,b)

3- saddle point at (a,b) if $f_{xx}f_{yy}-f_{xy}^2<0$ at (a,b).

4- Test fails if $f_{xx}f_{yy}-f_{xy}^2=0$ at (a,b)

* Critical point :- a point where $f_x, f_y = 0$ or one or both of f_x & f_y does not exist.

- Extreme values are critical points or boundary points

→ Saddle point is similar to the inflection point in single variable function

* When you find a critical point or Boundary point you need to check if it's min, max

→ use second derivative test

Back to Example 5