Chapter 15 :

15.1: Fubini's Theorem

$f(x,y)$ is continous Region R: $a \le x \le b$, $c \le y \le d$

Then:-

$$\text{Volume} = \iint_R f(x,y)\,dA = \int_c^d \int_a^b f(x,y)\,dx\,dy$$

$$\text{or} = \int_a^b \int_c^d f(x,y)\,dy\,dx$$

15.2: Double integrals over general form

$$V = \iint_R f(x,y)\,dA$$

V: Volume of solid enclosed betwe R and surfac $z = f(x,y)$

$$dA = dxy = dy\,dx$$

شرائح عرضية ← dxy ، شرائح طولية ← $dy\,dx$

• إذا بدنا نوجد حدود التكامل نعمل شرائح طولية أو شرائح عرضية

Steps to solve:

1- choose order $dy\,dx$ or $dx\,dy$

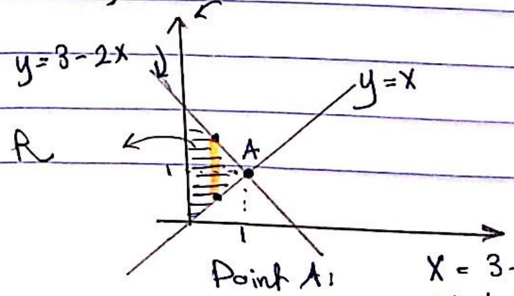
2- Draw limits Given

Ex:- find $\iint_R dy\,dx$, $\iint_R dx\,dy$ of Region bounded by:

$y = 3-2x$, $y = x$, $x = 0$

• If we used : $dy\,dx$

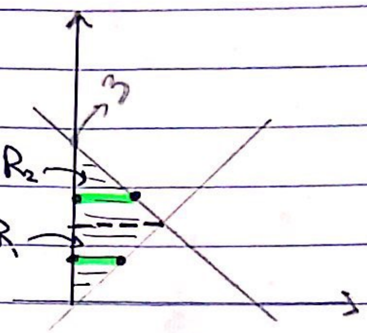
$$V = \int_0^1 \int_x^{3-2x} dy\,dx$$



Point A: $x = 3-2x$

$x = 1$

• If we used : $dx\,dy$



$$V = \int_0^1 \int_0^y dx\,dy + \int_1^{3/2} \int_0^{\frac{3-y}{2}} dx\,dy$$

$$= \int_0^1 y\,dy + \int_1^{3/2} \frac{3}{2} - \frac{y}{2}\,dy$$

$$= \frac{y^2}{2}\Big|_0^1 + \left[\frac{3}{2}y - \frac{y^2}{4}\right]_1^{3/2}$$

We will need to split it

$$= \frac{1}{2} + \left(\frac{3}{2} \times \frac{3}{2} - \frac{9}{4}\right) - \left(\frac{3}{2} - \frac{1}{4}\right)$$

$$= \cancel{1 + \frac{27}{16} - \frac{4\times 5}{4\times 4}} \qquad \frac{1}{2} + \frac{9}{4} - \frac{5}{4}$$

$$= \cancel{\frac{1\times 16}{1\times 16}} \quad \frac{7}{16} = \qquad \frac{1}{2} + 1 = \frac{3}{2}$$

15.3: Area by double integration

$$A = \iint_R dA$$

A: the area of a closed bounded plane region R

where $dA = dx\,dy = dy\,dx$

*** Average value:**

$$\text{Average value of } f \text{ over } R = \frac{1}{\text{area of } R} \iint_R f \, dA$$

15.4: Double integrals in Polar form:-

In polar form: $dA = r\,dr\,d\theta$

The area of a closed and bounded region R in the polar coordinate is

$$A = \iint_R r\,dr\,d\theta$$

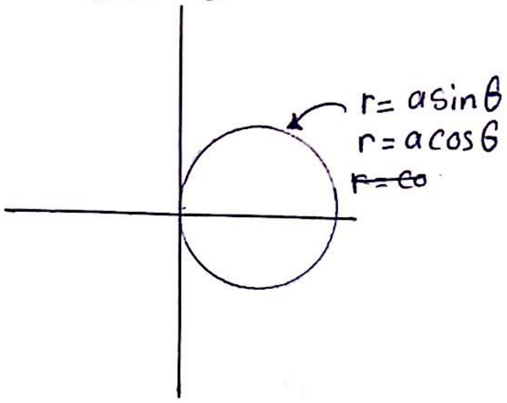
Remember:-

$x = r\cos\theta$ $\quad y = r\sin\theta$

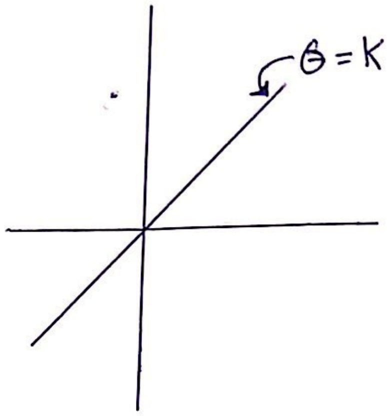
$x^2 + y^2 = r^2$ $\quad \tan\theta = \frac{y}{x}$

Famous graphs in polar form:-

circles:-



line:-



cardioid:-