15.5 Triple integrals in Rectangular Coordinates

* Volume of a region in space :-

$$V = \iiint dV$$

→ In cartesian:

$$dV = dz\,dx\,dy$$

عشان نوجد حدود التكامل ، أول order :- هو dz نباخذ
قطعة طولية موازية للـ z axis و بنشوف من وين تبلش و وين
تنتهي. بعدين بنسقط الشكل على xy plane و بنعمل زي التكامل
الثنائي

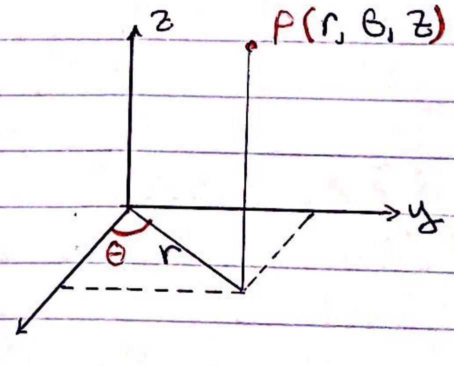
* Average value of a function in space

$$\text{Av of } F_{\text{(over D)}} = \frac{1}{\text{Volume of D}} \iiint_D F\,dV$$

+15.7 from here

→ In cylindrical coordinates

$$dV = r\,dz\,dr\,d\theta$$



• نباخذ قطعة طولية موازية لـ z
بعدين بنعمل مسقط للمنطقة على xy
و نباخذ شعاع من بداية المنطقة لآخرها
ومن وين لوين تمتد المنطقة (ربع أول ، $0 \to \frac{\pi}{2}$
ربعين ؛ $0 \to \pi$
⋮)

Transformation from cartesian to cylindrical.

$x = r\cos\theta$ $\qquad$ $r^2 = x^2 + y^2$

$y = r\sin\theta$ $\qquad$ $\tan\theta = \dfrac{y}{x}$

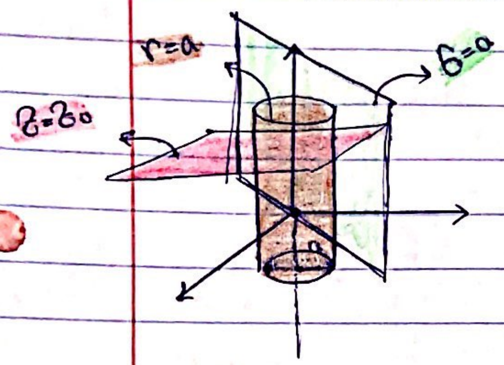
$z = z$

- Remember:

$\underline{r = a}$ cylinder, radius a

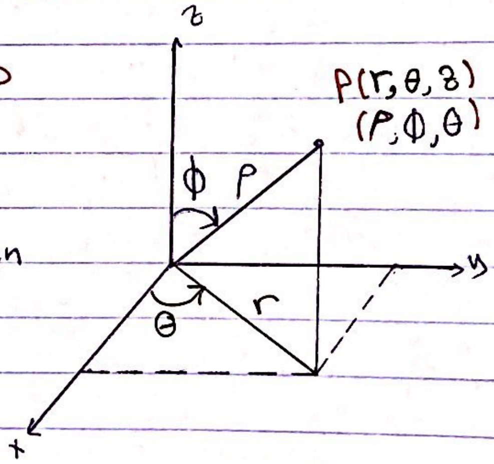
$\theta = a$ plane containing z-axis

$z = z_0$ Plane perpendicular to the z-axis



→ In spherical coordinates

- ϕ is the angle that ρ makes with $z+$
- θ same in cylindrical
- ρ distance from origin to point



Transformation from cartesian or cylindrical to spherical

$r = \rho\sin\phi$ $\qquad$ $x = r\cos\theta = \rho\sin\phi\cos\theta$

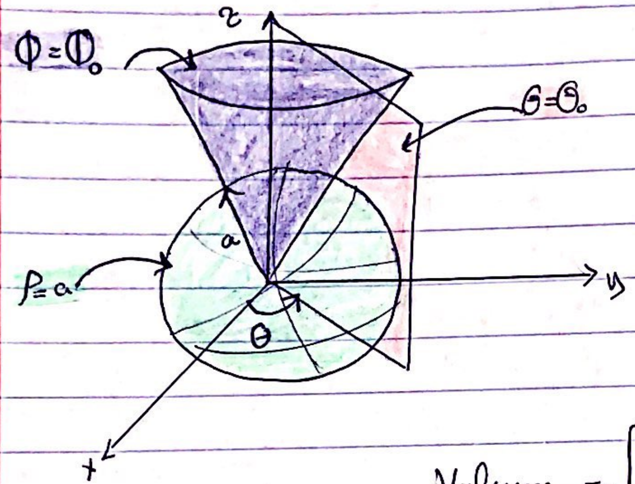
$z = \rho\cos\phi$ $\qquad$ $y = r\sin\theta = \rho\sin\phi\sin\theta$

$\rho = \sqrt{x^2+y^2+z^2} = \sqrt{r^2+z^2}$

Remember: $\rho = a$ Sphere at the origin with radius a

$\phi = \phi_0$ cone with it's vertex lying at the origion ~~at~~ and it's axis on the z axis

$\theta = \theta_0$: half plane containing z-axis and makes an angle θ_0 with the positive x-axis



$$\text{Volume} = \iiint \rho^2 \sin\phi \; \underline{d\rho}\, d\phi\, \underline{d\theta}$$

بنرسم شعاع من الصفر لآخر سطح في الرسمة والحدود بتكون من دخوله لخروجه

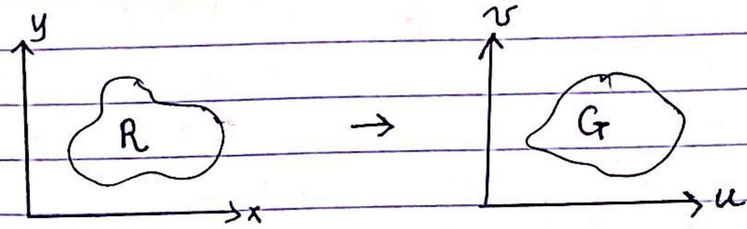
بأي ارباع موجودة الرسمة

هنشوف الرسمة عملت زاوية مع الـ z axis

15.8. Substitutions in multiple integrals

Double integrals

$$\iint_R f(x,y)\,dx\,dy = \iint_G f(g(u,v),\ h(u,v))\,J\ du\,dv$$



Steps to transform :-

1- transformation $x = g(u,v)$, $y = h(u,v)$ is Given by the question

2- we need to find new integration Limits
put values of x in : $x = g(u,v)$ and find v, u in each case
Same for y

3- find J :

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

4- Draw region G and find $dv\,du$ or $du\,dv$

Triple integrals

$$\iiint_R f(x,y,z)\,dx\,dy\,dz \Rightarrow \iiint_G H(u,v,w)\,J\,du\,dv\,dw$$

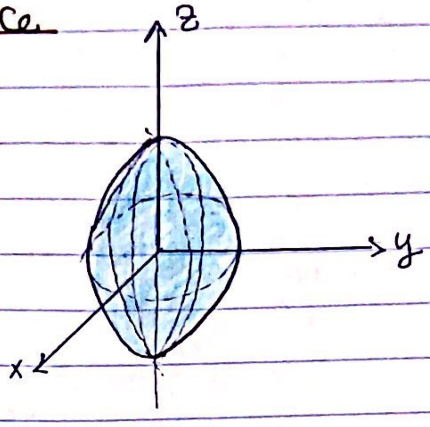
$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix}$$

Same way as Double integrals

Surfaces in Space

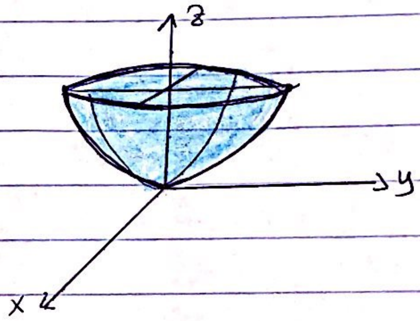
Ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



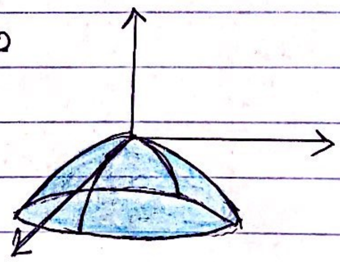
Paraboloid

$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$



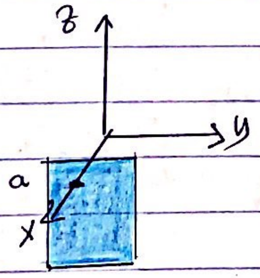
If x^2, y^2 were negative

Ex. $z = -x^2 - y^2$

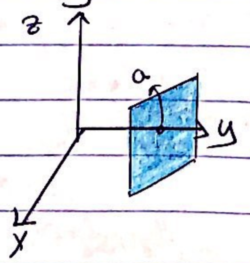


planes

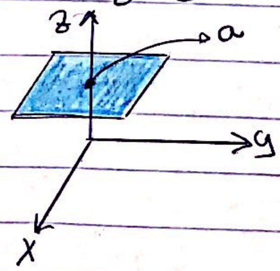
$x = a$



$y = b$



$z = c$



- How to find the equation of a plane

- you need three points on a vector and a point / a normal

→ If you have three points O, A, B

$O = (x_0, y_0, z_0)$

Find $OA = \vec{u}$

$OB = \vec{V}$

$n = \vec{u} \times \vec{V} = \left| \quad \right|$

$n = a\hat{i} + b\hat{j} + c\hat{k}$

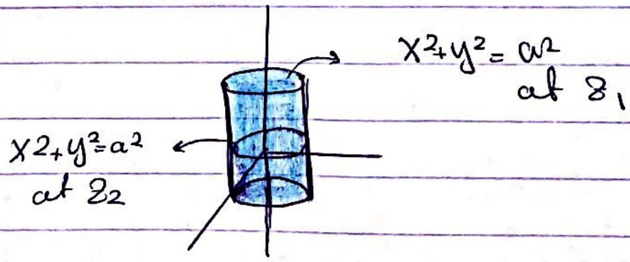
eq of plane : $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

→ If you have a point and a vector (normal)

Cylinder

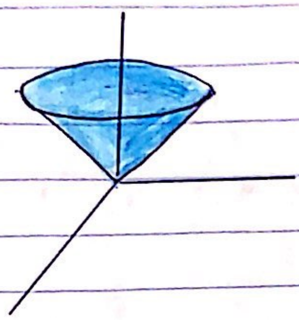
$x^2 + y^2 = a^2$ a radius , any value of z

(in cylindrical $r = a$)



Cone

$z = \sqrt{x^2 + y^2}$

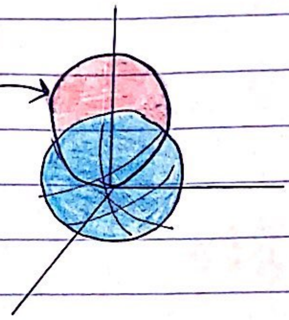


(In spherical $\phi =$)

$\frac{\pi}{4}$ = هاي الزاوية

Sphere

$z^2 + x^2 + y^2 = a^2$



(If in spherical $\rho = a\cos\phi$)

→ shift

• plane $z = x$