16.1: line integrals

- If f is defined on a curve C given parametrically by $r(t) = g(t)\hat{i} + h(t)\hat{j} + k(t)\hat{k}$, $a \le t \le b$

then line integral $= \int_C f(x,y,z)\,ds$

$$= \int_a^b f(g(t), h(t), k(t))\,|r'(t)|\,dt$$

$\rightsquigarrow r'(t) = \frac{dr(t)}{dt}$

* to find $r(t)$:- ① line between two points P, Q

$P(P_x, P_y, P_z)$

$PQ = (x_1)\hat{i} + (y_1)\hat{j} + (z_1)\hat{k}$

then $x = P_x + x_1 t$

$y = P_y + y_1 t$

$z = P_z + z_1 t$

② line through $P(P_x, P_y, P_z)$ and parallel to vector V

$V = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$

$x = P_x + V_x t$, $y = P_y + V_y t$

, $z = P_z + V_z t$

③ circle $C: x^2 + y^2 = a^2$

$r(t) = a\cos t\,\hat{i} + a\sin t\,\hat{j}$