Linear Final $\frac{26}{40}$

Birzeit University
Math. & Comp. Science Dept.
Linear Algebra

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Math 234 Final Exam

First Semester

Student Name:________________________ Number:________ Section: ____

Question One: 40 points) Circle the MOST correct statement

1. If the coefficient matrix of a system of linear equations is

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

then

(a) the system is consistent.

(b) if the system has a solution, it is unique.

(c) if the system has a solution, it has infinitely many solutions.

(d) the system is inconsistent.

2. If $A = (\vec{a}_1, ..., \vec{a}_n) \in R^{m\times n}$ where $\vec{a}_j$ denotes the j^{th} column of A, then $A\vec{x} = \vec{b}$ is consistent if and only if

(a) the system has a solution.

(b) $\vec{b} \in \text{Span}(\vec{a}_1, ..., \vec{a}_n)$.

(c) $\text{rank}(A|\vec{b}) = \text{rank}(A)$.

(d) all of the above.

3. If $A \in R^{n\times n}$ such that $A^n = I$, then

(a) $A^{-1} = A^{n-1}$.

(b) $\boxed{\det(A) = I}$.

(c) any $\vec{x} \in R^n$ is an eigenvector of A^n.

(d) all of the above.

$\lambda = 1, 1, 1, 1$

$A^n - \lambda^n I X = 0$

$(I - I)X = 0$

$0 \cdot X = 0$

$I - I = 0$ $X = 0$

4. Let $A = \frac{1}{4}\begin{bmatrix} 4 & -2\sqrt{3} & 0 \\ \sqrt{12} & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$.

$A = \frac{1}{4}\begin{bmatrix} 2 & -2\sqrt{3} & 0 \\ \sqrt{12} & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

(a) Is A non-singular? explain.

(b) If A^{-1} exists , Find A^{-1}.

(c) Find the rnak(A). $= 3$

5. Let $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 4 & 2 & 0 \\ 0 & 0 & 5 & 3 \end{bmatrix}$

(a) Find $det(A)$. $= 6$

(b) Find $\underline{C(X)}$.

(c) Find $N(A)$ and dimension of $N(A)$. $= 0$

dim $= 0$

$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{12}}{4}\right) = \frac{6}{8}$

$= \left(\frac{\sqrt{36}}{8}\right)$

$\frac{1}{4} + \frac{6}{8} = \frac{8+24}{32} = \frac{32}{32} = 1$

$A = \begin{pmatrix} 1 & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$-\frac{\sqrt{3}}{2} \times -\frac{\sqrt{3}}{2}$

$\frac{3}{4} + \frac{4}{4} = \frac{7}{4}$

$= \begin{pmatrix} 1 & -\frac{\sqrt{3}}{2} & 0 \\ 0 & \frac{7}{4} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\begin{matrix} 1 & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$

$|A| = \frac{7}{4} \neq 0$

$A \to$ nonsingular

$\begin{matrix} 1 & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 7 & \end{matrix}$

$\frac{4}{7}\begin{pmatrix} \frac{7}{4} & 0 & 0 \\ \frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & -\frac{7}{4} \end{pmatrix}$

$\begin{pmatrix} 1 & 0 & 0 \\ \frac{2\sqrt{3}}{7} & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

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Birzeit University
Math. & Comp. Science Dept.
Final Math. 234 Fall

Dr. Mohammad Saleh

Name:

Q1:(20 Points) True or false:

T (a) If A, B are invertible matrices then AB is invertible.

T (b) A homogeneous system might have finitely many solutions.

F (c) If the system $AX = B$ has more than one solution then A is invertible.

T (d) If A, B are two matrices and AB is invertible then A and B are invertible.

T (e) If A, B are two square nonzero matrices and $AB = 0$ then A and B are not invertible.

(f) Let A be a square and invertible $n \times n$ matrix. Then $|adj A| = |A|^n$.

T (g) Let A be a square $n \times n$ matrix. Then $|3A| = 3^n|A|$.

F (h) If A is a symmetric and skew symmetric then A must be a zero matrix.

F (i) If A, B are symmetric then AB is symmetric.

T (j) If all entries of the main diagonal of a square martix A are zeros then A is not invertible.

~~(k)~~ every metric space is a normed space.

~~(l)~~ every normed space is an inner product space.

T (m) if x_0 is a solution of the nonhomogeneous system $AX = B$ and x is a solution of the homogeneous system $AX = 0$. Then $x + x_0$ is a solution of the nonhomogeneous system $AX = B$.

F (n) If Wronskain$(f_1, \ldots, f_n) = 0$ then $f_1, \ldots, f_n$ are linearly dependent.

T (o) If A, B are similar then $p_A(\lambda) = p_B(\lambda)$.

F (p) If A is $n \times n$ and diagonalizable, then A has n different eigenvalues.

T (q) If 0 is an eigenvalue of A then A is not invertible.

F (r) If A is diagonalizable then A is diagonal.

T (s) If A is $n \times n$ and has n linearly independent eigenvectors, then A is invertible.

~~(t)~~ If A is 2×2, $tr(A) = 5$, and 2 is an eigenvalue of A, then 3 is an eigenvalue of A.

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Q2 : (22points) Circle the most correct answer

nonsing

(1) Let A be invertible. Then

(a) if A is symmetric then A^{-1} is symmetric
(b) If A is triangular then A^{-1} is triangular
(c) If A is diagonalizable then A^{-1} is *diagonalizable*
(d) All of the above

d

(2) If u, v are orthogonal vectors then

(a) $||u + v|| = ||u|| + ||v||$
(b) $||u|| \cdot ||v|| = || < u, v > ||$
(c) $|| < u, v > || = 1$
(d) none

(3) Define $< f, g > = \int_0^1 f(x) g(x) dx$ for every $f, g \in C[0,1]$, then $||x|| =$

(a) $\frac{1}{2}$
(b) $\frac{1}{\sqrt{2}}$
(c) $\frac{1}{\sqrt{3}}$
(d) $\frac{1}{4}$

(4) One of the following sets of vectors is l.d.

d

(a) $< 1, 1, 1 >, < 2, 3, \ldots$
(b) $< 1, 1, 3 >, < 2, \ldots, 4 >, < 1, 0, 0 >$
(c) $1, x, x^2$
(d) $< 1, 0, 1 >, < 2, 3, 5 >, < 6, 4, 3 >, < 3, 2, 5 >, < 5, 0, 0 >$

(5) One of the following is not a basis for the corresponding space

(a) $< 1, 1, 1 >, < 2, 3, 1 >, R^2$
(b) $< 1, 0, 1 >, < 2, 3, 5 >, < 6, 4, 3 >, < 3, 2, 5 >, < 5, 0, 0 >, R^3$
(c) x, x^2, x^3, P_3
(d) *all of the above*

(6) The dimension of the column space of $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{bmatrix}$ is

$$\begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -6 & 1 \end{pmatrix}$$

(a) 3
(b) 4
(c) 2
(d) 1

(7) If A is $n \times n$ invertible matrix then

(a) $Rank(A) = n$
(b) $Nullity(A) = n$
(c) $Nullity(A) = n - 1$
(d) $Nullity(A) = n^2$

(8) If an $n \times n$ matrix A has only one eigenvalue λ then dimension of the eigenspace corresponding to λ is.

(a) n.

(b) ≥ 1.

(c) $= 1$.

(d) 0

(b)?

(9) If an 4×4 matrix A has 1,-1,3,5 as its eigenvalues then $det(A) =$

$(1)(-1)(3)(5) = -15$

(a) 8

(b) 15

(c) -15

(d) -8

(10) Let $A = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$ then the eigenvalues of A^{100}

(a) ~~1, 2~~

(b) $1, 2^{100}$

(c) 2^{100}

(d) *none*

$\boxed{1 \times 2}$

$\begin{pmatrix} 1-\lambda & 3 \\ 0 & 2-\lambda \end{pmatrix}$

1, 2

(11) The nullity of $[1\ 1\ 1\ 1]$ is

$\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

(a) 1

(b) 2

(c) 3

(d) 4

~~$-6-6=-12$~~

Q5 (20points) Let $A = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$.

a) Find the eigenvalues and eigenvectors of A.

$\begin{pmatrix} 1-\lambda & 1 & 2 \\ 0 & 2-\lambda & 1 \\ 0 & 0 & 3-\lambda \end{pmatrix}$

$= (1-\lambda)(2-\lambda)(3-\lambda) = 0$

$\lambda_1 = 1$ $\lambda_2 = 2$ $\lambda_3 = 3$

$E(1)$

b) Is A diagonalizable, if yes find a matrix P that diagonalize A.

$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$,

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MATH. 234, FINAL EXAM, FIRST SEMESTER

Dr. Ayman Abuhijleh (Sections 1 and 4)
Dr. Jawad Abuhlail (Section 5)
Dr. Alaeddin Elayyan (sections 2 and 3)
Dr. Hasan Yousef (Section 6)

Name——————————, ID. Number————————————, Score

QUESTION 1. *(points)* **(Write Down True or False)**

(1) *If A and B a $n \times n$ matrices, then $det(AB) = det(A)det(B)$* (T)

(2) *If A and B $n \times n$ nonsingular matrices, then $A + B$ is a nonsingular matrix* (F)

(3) *If A is an n n matrix and $b \in R^n$ such that $AX = b$ has no solution, then A is singula.* (T)

(4) *If A and B are $n \times n$ matrices and A is row equivalent to B, then $det(A) = det(B)$* (F)

(5) *It is possible that $x^2 - x$ be the characteristic polynomial of a nonsingular 2×2 matrix.* F

(6) *If A is an $n \times n$ matrix and $det(A) = 0$, then $AX = 0$ has infinitely many solutions* T

(7) *It is possible to have $v_1, v_2, v_3 \in R^4$ such that $span\{v_1, v_2, v_3\} = R^4$* (F)

(8) *Let V be the vector space of all 2×3 matrices. Then $dim(V) = 5$.* (F)

(9) *If A is a 2×2 nonzero singular matrix, then $Nullity(A) = 1$* ( )

(10) *If A and B are $n \times n$ matrices and A is row equivalent to B, then A is singular if and only if B is singular* (T)

(11) *If A is an $n \times n$ matrix and a, b are eigenvalues of A, then $a + b$ is an eigenvalue of A.* (—F—)

(12) *If A is an $n \times n$ matrix and a, b are two distinct eigenvalus of A and $v \in Nul(A - aI) \cap Nul(A - bI)$, then v is the zero vector.* (———)

(13) *If A is a singular $n \times n$ matrix, then 0 is an eigenvalue of A.* (—T—)

(14) *If T is a linear transformation from R^4 to R^3, then there is a matrix A, 3×4, such that $T(v) = Av$ for every $v \in R^4$.* (—T—)

(15) *There is a linear transformation from R^4 to R^2 such that T is one to one.* (—F—)

(16) *If A is an $n \times n$ matrix such that $det(A) = det(-A)$, then n must be an even positive integer.* (—F—)

$T(v) = {}_{3\times4} \, v_{4\times1}$

v

(17) *If A, B are $n \times n$ nonzero matrices such that AB is the zero matrix, then both A, B are singular.* (—T—)

(18) *If A is a nonzero 3×2 matrix such that $Ax = 0$ has infinitely many solutions, then Nullity$(A) = 2$.*(———)

(19) *If A is 5×8 matrix such that Rank$(A) = 3$, then Nullity$(A) = 2$.*(—F—)

(20) *There is a matrix, say A, 6×9 such that Rank$(A) = 7$.* (—F—)

(21) *If A is a nonsingular $n \times n$ matrix such that $Av = 3v$ for some nonzero vector $v \in R^n$, then $A^{-1}v = -3v$.* (———)

(22) *If A is an 7×4 matrix and Rank$(A) = 4$ and AB is the zeo matrix for some matrix B, 4×2, then B must be the zero matrix.* (———)

$n=2$

rank$=0$

(23) *If A, B are nonsingular $n \times n$ matrices, then A is row equivalent to B.* (—T—)

(24) *If A, B are $n \times n$ matrices and $B = 3A$, then $\det(B) = 3\det(A)$* (—F—)

(25) *If A, B are $n \times n$ matrices and A is row equivalent to B, then $A = PB$ for some nonsingular $n \times n$ matrix P.* (—F—)

(26) $\{x^2+1, x^2+x\}$ *is a basis of P_3.* (—F—)

(27) $dim(\{x^3+3, x^3+1, x^3+x\}) = 2$. (—F—)

(28) $S = \{f(x) \in P_3 : f(1) = 0 or f(0) = -1\}$ *is a subspace of P_3.* (—F—)

(29) $S = \{f(x) \in P_5 : f(x) = -f(x)\}$ *is a 3-dimensional subspace of P_5.* (———)

(30) *Given* $S = \{f(x) \in P5 : f(1) = f(-1)\}$ *is a subspace of P_5. Then* $\{1, x^2, x^4\}$ *is a basis of S.* (—F—)

QUESTION 2. *(30 points,* **Each = 1.5 points***)(***CIRCLE THE CORRECT ANSWER***)*

(1) *Let A and B be 3×3 matrices such that* $\det(A) = -1$ *and* $\det(B) = -2$. *Then* $\det(2AB^{-1}) =$

(a) *1* $\boxed{\text{(b) } 4}$ (c) *16* (d) *None*

(2) *Given* $S = \{A \in R^{4\times4} : A$ *is a diagonal matrix\}* *is a subspace of* $R^{4\times4}$. *Then* $dim(S) =$

(a) *2* (b *3* $\boxed{\text{(c) } 4}$ (d) *None*

(3) *Given* $S = \{f(x) \in P_9 : f(1) = f(-2) = 0\}$ *is a subspace of P_9. Then* $dim(S) =$

(a) *7* (b) *8* (c) *9* (d) *None*

(4) *If A is a 5×3 matrix and $b \in R^5$ such that $AX = b$ has exactly one solution. Then Nullity$(A) =$*

(a) *0* (b) *at least one* (c) *3* (d) *None*

rank$=3$

(5) *Let* $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. *Then the eigenvalues of A are*

(a) *1 and -1* (b) *0, 1, and -1* (c) *-1, 2, and 0* $\boxed{\text{(d) None}}$

$$\begin{matrix} 0-\lambda & 0 & 0 \\ 1 & -\lambda & 1 \\ 0 & 1 & -\lambda \end{matrix}$$

$(-\lambda)(-\lambda)(-\lambda)^{20}$

$(1-\lambda)(2-\lambda)(1-\lambda)(2-\lambda)$

$(1-\lambda)^2(2-\lambda)^2$

$-1(\lambda-1)$

(6) *If A is a 4×4 matrix and A is similar to* $\begin{bmatrix}1&2&0&8\\0&2&1&-4\\0&0&1&5\\0&0&0&2\end{bmatrix}$, *then the characteristic polynomial of A is*

$\boxed{\text{(a)}}$ $(x-1)^2(x-2)^2$ (b) $(x+1)^2(x+2)^2$ (c) $(x-1)(x-2)$ (d) *None*

(7) *If -2 is an eigenvalue of a 4×4 matrix A, then an eigenvalue of $4A$ is*

(a) *1/2* (b) *-1/2* (c) *-8* (d) *16*

$4(-2-\lambda)$

$-8-4\lambda=0$

(8) *Let* $A=\begin{bmatrix}1&-1\\1&1\end{bmatrix}$, $B=\begin{bmatrix}-2&2\\-2&-2\end{bmatrix}$, *and* $C=\begin{bmatrix}1&0\\1&1\end{bmatrix}$. *Then a basis for Span$\{A,B,C\}$ is*

$\boxed{\text{(a) } \{A,C\}}$ (b) $\{A,B,C\}$ (c) $\{A,B\}$ (d) *None*

$\begin{pmatrix}1&-2&1\\-1&2&0\\1&-2&1\\1&-2&1\end{pmatrix}$

(9) *If v_1,v_2 are independent in R^3 and v_3 is a (nonzero) element of R^3 such that $v_3=2v_1+-3v_2$. Then*

(a) *dim Span$\{v_1,v_2,v_3\}=2$* (b) *$\{v_1,v_3\}$ are independent* (c) *$\{v_2,v_3\}$ are independent* (d) *a, b, and c are correct statements*

(10) *ONE of the the following is not a vector space*

(a) $V=\{f(x)\in P_9 : f(0)=0\}$ (b) $V=span\{e^x, sinx, tanx\}$ (c) $V=$ *all upper triangular 5×5 matrices* (d) $V=\{f(x)\in P_{13} : f(0)=1\}$

bord

$(\lambda+2)$

$\begin{pmatrix}1&-2&1\\0&\boxed{0}&0\\0&0&0\\0&0&0\end{pmatrix}$

(11) *Given* $A=\begin{bmatrix}2&-3\\2&-5\end{bmatrix}$ *is similar to a diagonal matrix D. Then $D=$*

(a) $\begin{bmatrix}1&0\\0&-4\end{bmatrix}$ (b) $\begin{bmatrix}2&0\\0&-5\end{bmatrix}$ (c) $\begin{bmatrix}-2&0\\0&5\end{bmatrix}$ (d) $\begin{bmatrix}-1&0\\0&4\end{bmatrix}$

$(2-\lambda)(-5-\lambda)$

$+6$

$-10-2\lambda+5\lambda+6$

$5\lambda^2-2\lambda-4$

0

(12) *Given A is a 2×2 matrix and 2, 1 are eigenvalues of A. Then the characteristic polynomial of A^{-1} is*

$\boxed{\text{(a)}}$ $x^2-(3/2)x+1/2$ (b) $x^2+(1/3)x+1/2$ (c) x^2-3x+2 (d) $1/(x^2-3x+2)$

$x_3=0$

$x_2=\alpha$

$x_1=\alpha$

(13) *Let $T: R^3 \Longrightarrow R^2$ such that $T(x_1,x_2,x_3)=(x_1+x_2, x_2+x_3)$. Given $B=\{(1,2,1),(0,2,-1),(-1,-2,3)\}$ is a basis for R^3 and $H=\{e_1,e_2\}$ is the standard basis for R^2. Then the matrix representation of T with respect to B and H is*

(a) $\begin{bmatrix}1&0&-1\\2&2&-2\\1&-1&3\end{bmatrix}$ (b) $\begin{bmatrix}3&2&-3\\3&1&1\end{bmatrix}$ (c) $\begin{bmatrix}3&3&2\\1&1&-3\end{bmatrix}$ (d) *None*

(14) *Given A is a $4\times m$ matrix, B is a $m\times 4$ matrix, and $C=AB$. Suppose that C is nonsingular (observe that C is a 4×43 matrix). Then*

(a) $m<4$ $\boxed{\text{(b) } m=4}$ (c) $m<4$ (d) $m\geq 4$.

4×4

$\frac{2\times 1}{2\times 1} \to \frac{1}{2}$

$\frac{1}{2}$

$\frac{3}{2}$

$\det(A)=2 \to \frac{1}{2}$

$tra(A)=3 \to -10-2\lambda+5\lambda+\lambda^2+6$

$-4+3\lambda+\lambda^2$

x^2-bx+c

$x^2-\frac{3}{2}x+\frac{1}{2}$

$\lambda^2+3\lambda-4$

$(\lambda+4)(\lambda-1)$

$\lambda=1$

(15) *Let* $A = \begin{bmatrix} 2 & 0 & 0 & -2 \\ -4 & 0 & 0 & 4 \\ 0 & 3 & 0 & 0 \end{bmatrix}$. *A basis for the row space of A is*
(a) $\{(2,0,0,-2),(-4,0,0,4),(0,3,0,0)\}$ (b) $\{(2,0,0,-2),(0,0,1,0)\}$
(c) $\{(1,0,0,-1),(0,1,1,0\}$ (d) $\{(2,0,0,-2),(0,3,0,0)\}$

(16) *Let* $A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix}$. *A basis for N(A) is*
(a) $\{(1,0,-1,1),(0,1,2,-1)\}$ (b) $\{(1,0),(0,1)\}$ (c) $\{(0,-2,1,1),(1,-3,1,0)\}$
(d) $\{(1,-2,1,0),(-1,1,0,1)\}$

(17) *The solutions for*
$x_1 + x_2 + x_3 + x_4 + x_5 = 2$
$x_1 + x_2 + x_3 + 2x_4 + 2x_5 = 3$
$x_1 + x_2 + x_3 + 2x_4 + 3x_5 = 2$

(a) $\{(x_1, x_2, x_3, 2x_1, -x3) : x_1, x_2, x_3 \in R\}$ (b) $\{(2, -1, 1+x_3, x_3, 0, -2) : x_3 \in R\}$
(c) $\{(1-x_2-x_3, x_2, x_3, 2, -1) : x_2, x_3 \in R\}$ (d) $\{(x_2+x_3, x_2, x_3, 2x_2, -x_3) : x_2, x_3 \in R\}$

$X_4 = 0$ $X_5 = 1$

(18) *Let* $A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 0 & (c-3) \\ 0 & 3 & 10 \end{bmatrix}$. *The values of c that make $AX = b$ has a solution for every $b \in R^3$ are*
(a) *All real numbers* (b) ~~*All real numbers except 3*~~ (c) *0* (d) *All real numbers except 0.*

(19) *Given* $A = \begin{bmatrix} 2 & 4 & 1 \\ 0 & 0 & 2 \\ 0 & -1 & 2 \end{bmatrix}$. *The (1, 3)-entry of A^{-1} is*
(a) *-2* (b) *2* (c) *0* (d) *None*

(20) *Given v_1, v_2 are independent in R^3. Then*
(a) *If $v_3 \in span\{v_1, v_2\}$, then it is possible that $\{v_1, v_2, v_3\}$ form a basis for R^3* (b) *It is possible that $(0,0,0) \notin Span\{v_1, v_2\}$*
(c) *If Span $\{v_1, v_2, v_3\} = R^3$, then $v_3 \notin Span\{v_1, v_2\}$* (d) *a, b, and c are correct statements*

$X_3 = \alpha$
$X_4 = \beta$
$X_2 = \beta$
$X_4 = \alpha$
$X_3 = \alpha$
$X_1 = -\alpha - \beta$

$-X_2 = 2X_3 - X_4$
$X_2 = -2\alpha + \beta$

$\alpha \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

$\alpha \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$

27/28

Birzeit University
Math. & Comp. Science Dept.
Math. 234

Dr. Ayman Abu- Hijleh

Summer

Final Exam

Section: —–

Student Name: Badee' Awwad Number:

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1. Let $A = \begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix}$.

$A = \begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix}$

$\begin{pmatrix} 2-\lambda & -3 \\ 1 & -2-\lambda \end{pmatrix}$

$(2-\lambda)(-2-\lambda)+3 = 0$

$-4 - \cancel{2\lambda} + \cancel{2\lambda} + \lambda^2 + 3 = 0$

$-1 + \lambda^2 = 0$

$\boxed{\lambda = \pm 1}$

(a) Find the eigenvalues of A.

(b) Explain, why A is diagonalizable.

(c) Find a diagonal matrix D and a matrix Q s.t. $Q^{-1}AQ = D$.

(d) Find A^{103}

2. Let $T : R^2 \to R^3$, given by $T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3x_1 \\ 0 \\ 2x_2 \end{pmatrix}$ is a L.T. Also, given $B_2 = \{(3,2),(1,6)\}$ is a basis for R^2, and $B_3 = \{(2,0,4),(-4,0,2),(0,30)\}$ is a basis for R^3.

$3x_1$
0
$2x_2$

$x_1\begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$

(a) Find the matrix A representing T in terms of B_3 and B_2.

(b) Find $\left[T\begin{pmatrix} 5 \\ 10 \end{pmatrix}\right]_{B_3}$

(c) Find the dimension of $Range(T)$ and the dimension of $Ker(T)$.

2 1

3. Let $B = \{10, 3x+1, 5x^2+2x-1\}$

$\begin{pmatrix} 3x_1 \\ 0 \\ 2x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

(a) Show that B is a basis for P_2.

(b) Find the transition matrix from B to $\{1, x, x^2\}$.

(c) Find the transition matrix from $\{1, x, x^2\}$ to B.

(d) Write $P(x) = 3x^2 + 17x + 33$ as a linear combination of the elements in B.

$\begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix}$

$x_3 = \alpha$
$x_2 = 0$
$x_1 = 0$

27
4) 108
8
28

27
17
100
85
15

4. If A and B are two matrices such that $(AB)^t = A^tB^t$, then

(a) $AB = BA$.

(b) A is a square matrix.

(c) B is a square matrix.

(d) Both (a) and (b) are correct.

5. If both U and V are subspaces of a vector space W, then

(a) $U \cap V$ is a subspace of W.

(b) $U + V$ is a subspace of W.

(c) the union need not be a subspace of W.

(d) all of the above.

6. One of the following statements is always true:

(a) If $f,\ g \in C(-\infty, \infty)$ such that $W[f,g](x) = 0$ for all $x \in (-\infty, \infty)$, then f and g are linearly dependent.

(b) A subset S of a vector space V is linearly independent if no element in S can be written as a linear combination of the other elements of S.

(c) Any element in a linearly dependent set can be written as a linear combination of the other elements in the set.

(d) all of the above.

7. The statement *An $n \times n$ matrix A is nonsingular* is equivalent to

(a) $\vec{0} \in N(A)$.

(b) $\text{Span}(\vec{a}_1, ..., \vec{a}_n) = R^n$.

(c) $\text{rank}(A) + \text{nullity}(A) = n$.

(d) all of the above.

8. Which of the following transformations $T : R^2 \longrightarrow R^2$ is not linear?

(a) Reflection: $T(\vec{x}) = -\vec{x}$.

(b) Contraction: $T(\vec{x}) = \alpha\vec{x}, 0 < \alpha < 1$.

(c) Expansion: $T(\vec{x}) = \alpha\vec{x}, 1 < \alpha$.

(d) Translation: $T(\vec{x}) = \vec{x} + \vec{a}$

9. In R^2 with $||\vec{x}||_1 = |x_1| + |x_2|$, $||\vec{x}||_2 = \sqrt{x_1^2 + x_2^2}$, and $||\vec{x}||_\infty = \max\{|x_1|, |x_2|\}$,

(a) $||\vec{x}||_1 \leq ||\vec{x}||_2 \leq ||\vec{x}||_\infty$.

(b) $||\vec{x}||_1 \leq ||\vec{x}||_\infty \leq ||\vec{x}||_2$.

(c) $||\vec{x}||_\infty \leq ||\vec{x}||_1 \leq ||\vec{x}||_2$.

(d) $||\vec{x}||_\infty \leq ||\vec{x}||_2 \leq ||\vec{x}||_1$.

$-T_u = T_u + \lambda q - \ldots$

Birzeit University

Math. Dept.

Math 434: Advanced Linear Algebra

[redacted]

M. Saleh

LA

Final Exam

Student Name:____________________________Number:________ Section __ First Semester 2012/2013

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1. (a) Prove that the eigenvectors corresponding to distinct eigenvalues are linearly independent

   (b) Prove that the the generalized eigenvectors in a chain are linearly independent

   (c) Let $L : C^2 \to C^2$ be a linear operator defined by $L(z, w) = (z + w, z - w)$. Find a basis for C^2 so that the matrix representation of L with respect to this basis is upper triangular.

   (d) Show that an orthogonal set of vectors is linearly independent

   (e) Show that a real symmetric matrix is diagonalizable

2. (a) Show that any linear operator over a finite dimensional complex space has an eigenvector

   $\beta_1(\lambda_1 - \lambda_k$

   (b) Show that any linear operator over a finite dimensional complex space has an upper triangular matrix representation

   (c) Define an inner product on R^n by $< x, y >= x^t y, x, y \in R^n$. Let A be an $n \times n$ matrix. Show that $||Ax||^2 =< x, A^T Ax >$, for every $x \in R^n$.

   (d) Let λ_1, λ_2 be distinct eigenvalues of A. Let x be an eigenvector of A corresponding to λ_1, y be an eigenvector of A^t corresponding to λ_2. Show that x, y are orthogonal vectors.

   $A \quad x \longrightarrow \lambda_1$
   $A^t \quad y \longrightarrow \lambda_2$

3. (a) Use Gram-Schmidt to find the distance between the point $(1, 1, 1)$ and the plane $x - y = 1$

   (b) Use Gram-Schmidt to find the formula for the distance between the point (x_0, y_0) and the line $ax + by = c$

   (c) Use Gram-Schmidt to approximate $sinx$ by a polynomial of degree 1. (use inner product on P_n to be $< f, g >= \int_0^1 fg dx$

   (d) Find the adjoint of the linear operator $L : R^3 \to R^2$ defined by $L(x, y, z) = (x - y, x + z - y)$
   with respect to the standard basis of R^3 and the basis $(1, 1), (1, 2)$ for R^2

10. If B is similar to A, then

(a) $\det(B) = \det(A)$.

(b) $\mathrm{tr}(B) = \mathrm{tr}(A)$.

(c) $\mathrm{rank}(B) = \mathrm{rank}(A)$.

(d) all of the above.

11. An $n \times n$ matrix A will never have a zero eigenvalue if

(a) $A^t = A$.

(b) $A^t = -A$.

(c) $A^t = A^{-1}$.

(d) all of the above.