Birzeit University
Department of Mathematics
math 243

First hour exam
Name : .... Key.

Fall 2018
Number:.........

Question# 1 (12%) Consider the following statement "If ab =0 then a=0 or b=0 "

a) Write the converse of this statement

If $a=0$ or $b=0$ then $ab=0$

b) Write the contrapositive of this statement

If $a\neq 0$ and $b\neq 0$ then $ab\neq 0$

c) Write the negation of this statement

$ab=0$ and $a\neq 0$ and $b\neq 0$

d) Prove this statement by usimg the direct proof

suppose $ab=0$ then
if $a=0$ then there is nothing to prove
if $a\neq 0$ then $\frac{1}{a}\neq 0$ and $\frac{1}{a}(ab)=\frac{1}{a}\cdot 0$ so $b=0$.

Question # 2 (9%):Let A={{2},5} , B={5 , 6 , 2} , C={5 , 2 , 7 } , find

a) $A\cap B = \{5\}$

b) $A-C = \{\{2\}\}$

c) $\rho(A) = \{\phi, \{\{2\}\}, \{5\}, \{\{2\}, 5\}\}$

Question #3(24%): Which of the following statements is true and which is false? Justify your answer

1)........ If $x \in A$, and $A \in B$ then $x \in B$

F

Example: $A = \{x\}$, $B = \{\{x\}\}$

2)........ $(\exists x \in \mathbb{Z})(\forall y \in \mathbb{Z})(x<y^2)$

T

proof: Take $x = -1$.

3)........ If $A \cap B = A \cap C$ then $B=C$

F

Example: $A = \{1, 2\}$, $B = \{2, 5\}$, $C = \{2, 7\}$

4)........ If $B \subset A$ then $A-B \neq \phi$

T

proof:- $B \subset A$ implies $\exists x \in A, x \notin B$.

$\Rightarrow x \in A - B$

$\Rightarrow A - B \neq \phi$.

5) $(\forall x \in \mathbb{R}^+)\ (\exists n \in \mathbb{N})\ (\frac{1}{n} < x)$

T

Let $x \in \mathbb{R}^+$, then if $x > 1$ take $n = 2$

then $\frac{1}{n} = \frac{1}{2} < x$.

But if $0 < x < 1$ then, there is $n \in \mathbb{Z}$

such that $1 < nx$, $n = \lceil \frac{1}{x} \rceil$ say. (least integer greater than or equal x.

and then $\frac{1}{n} < x$.

6)……… $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(\exists z \in \mathbb{R})(z > x + y)$

T

Let x, y be given then take $z = x+y+1$.

Question # 4 (21%):Prove the following

a)For any sets A,B,C .Prove that if $C \subseteq A \cup B$ and $B \cap C = \phi$ then $C \subseteq A$

Let $x \in C \Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in A$ since $B \cap C = \phi$.

b)For any sets A,B If $\rho(A) \subseteq \rho(B)$ then $A \subseteq B$

Two proofs:

① $A \in \rho(A) \Rightarrow A \in \rho(B)$ (since $\rho(A) \subseteq \rho(B)$)

$\Rightarrow A \subseteq B$.

② Let $x \in A \Rightarrow \{x\} \in \rho(A) \Rightarrow \{x\} \in \rho(B) \Rightarrow x \in B$

so $A \subseteq B$.

c) if x and y $\in \mathbb{R}$ and x $\neq 0$ and y $\neq 0$ then $x^2 + xy + y^2 > 0$

Let $x, y \in \mathbb{R} \Rightarrow x^2 + xy + y^2 = x^2 + xy + \frac{1}{4}y^2 + \frac{3}{4}y^2$

$x, y \neq 0$

$= (x + \frac{1}{2}y)^2 + \frac{3}{4}y^2$

> 0

since $(x + \frac{1}{2}y)^2 \geq 0$ and $\frac{3}{4}y^2 > 0$.

Question #5 (20%) for each $k \in \mathbb{N}$ let

$A_k = (-\frac{1}{k}, 2 + \frac{1}{k}]$, B=(-2,5) find

a) $\bigcup_{k=1}^{\infty} A_k = (-1, 3]$

b) $\bigcap_{k=1}^{\infty} A_k = [0, 2]$

c) $(\bigcup_{k=10}^{\infty} \overline{A_k}) = \mathbb{R} - [0, 2]$

d) $(\bigcup_{k=1}^{\infty} B \cap A_k) = (-1, 3]$