جامعة بيرزيت
BIRZEIT UNIVERSITY

MATHEMATICS DEPARTMENT
MATH234 -First Exam-
First Semester 2017/2018

88/100

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Section	Teacher	Time
1	Reema Sbeih	S,M,W 11:00 - 11:50
2	Hasan Yousef	S,M,W 09:00 - 09:50
3	Hani Kabajah	T,R 09:30 - 10:50
4	Khaled Altakhman	T,R 09:30 - 10:50
5	Hasan Yousef	S,M,W 11:00 - 11:50
6	Mahmoud Ghannam	S,M,W 12:00 - 12:50 ✓

(Q1) [32 points] Fill the blanks with True (T) or False (F)

✓ [F] (1) If A, B are $n \times n$ matrices and $AB = 0$, then either $A = 0$ or $B = 0$ $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

✓ [F] (2) If A is row equivalent to B, then $det(A) = det(B)$

✓ [F] (3) The product of two elementary matrices is an elementary matrix

✓ [T] (4) The inverse of an elementary matrix is an elementary matrix of the same type

A^{-1} is exist

F [T] (5) If A is an $\underline{n \times n}$ matrix and $Ax = b$ is $\underline{\text{consistent}}$ for some $b \in \mathbb{R}^n$, then A is nonsingular

✓ [T] (6) If A is a symmetric nonsingular matrix, then A^{-1} is also symmetric

✓ [F] (7) If A, B are $n \times n$ matrices such that $AB = 0$, then A and B are singular $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 3 & 6 \end{bmatrix}$ $|A||B| = 0$

✓ [T] (8) If the coefficient matrix of the system $Ax = 0$ is $\underline{\text{singular}}$, then the system has infinite number of solutions

يمكن توصيله لـ I   trivial so one solu nonsingular   product of elementary RREF = I

$n \times n$

✓ [T] (9) A square matrix A is $\underline{\text{nonsingular}}$ $\underline{\underline{\text{iff}}}$ its reduced row echelon form is the identity matrix ✱

✓ [T] (10) If $AB = AC$ and $det(A) \neq 0$, then $B = C$ $AB = AC \Rightarrow A^{-1}AB = A^{-1}AC$

✓ [T] (11) If A is a singular matrix, then A^T is also singular.

✓ [F] (12) If A, B are singular, then $A + B$ is also singular.

✓ [T] (13) Any two $n \times n$ nonsigular matrices are row equivalent. ✱

✓ [T] (14) If A is a singular matrix and U is the row echelon form of A, then $det(U) = 0$

A singular
At least one row
is zero

✓ [T] (15) If A is a 3×3 matrix such that $a_2 = a_3$, then $adjA$ is singular

✓ [T] (16) A homogeneous linear system $\underline{\underline{\text{can}}}$ have a nontrivial solution.

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$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

$\begin{matrix} 1 & \frac{1}{2} \\ 1 & 3 \end{matrix} \Rightarrow \begin{matrix} 1 & \frac{1}{2} \\ 0 & 3 - \frac{1}{2} \end{matrix} \quad \begin{matrix} 1 & \frac{1}{2} \\ 0 & \frac{5}{2} \end{matrix}$

$\begin{matrix} 1 & \frac{1}{2} \\ 0 & 1 \end{matrix} \quad \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$

(Q2) [36 points] Circle the correct answer

(1) If A is a $\underline{4 \times 4}$ matrix such that $a_2 + a_3 - a_4 = 0$, then

(a) A is singular

(b) A is nonsingular

(c) the system $Ax = 0$ has only the trivial solution

(d) None of the above

$Ax = b$ Homogeneous

$A \begin{bmatrix} 0 \\ 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

many solution or inconsistent] singular

one solution ⇒ nonsingular

(2) If A, B are 6×6 symmetric matrices, then one of the following matrices is always symmetric

(a) AB

(b) $AB - BA$

(c) $AB + A$

(d) $A - B$

$(AB)^T = B^T A^T = BA$

$BA - AB$

$BA + A$

$A^T - B^T = A - B$

(3) If A is an $n \times n$ matrix such that $AA^T = I$, then

(a) $A^T = A^{-1}$

(b) $(A^{-1})^T = A$

(c) $det(A) = \pm 1$

(d) All of the above

inverse

Proof $A^{-1} * A = I$

$|A||A|^{-1} = 1$

$|A| = |A|$

$AA^{-1} = I$

$\frac{|A|}{|A|} = |I|$

$\frac{1}{1} = 1$

$\frac{-1}{-1} = 1$

(4) If $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $det(A) = 5$, then $\begin{vmatrix} 2a & 2b & 2c \\ g & h & i \\ -d & -e & -f \end{vmatrix} =$

(a) 10

(b) -10

(c) 40

(d) -40

Row op 2 E^2

Row op 1 E^1

Row op 2

$E^3 = 1$

$E^1 = -1$

$5 * 2 * -1 * -1$

$\Rightarrow 10$

(5) If E is a 5×5 elementary matrix of type I , then

(a) $det(E) = 1$ ✗

(b) $det(adj E) = 1$ ✓

(c) $det(E^T) = 1$ ✗

(d) $det(E^{-1}) = 1$ same type

$E^{+1} = {}^{-}1$

$|adj E| = |E|^{n-1}$
$= |E|^4$
$= (^{-}1)^4 = 1$

(6) If $Ax = b$ is inconsistent, where A is an $n \times n$ matrix and $b \in \mathbb{R}^n$, then

no solution [singular]

(a) A is row equivalent to I ✗

(b) A is nonsingular ✗

(c) $Ax = 0$ has infinitely many solutions

(d) A is a product of elementary matrices ✗

(7) Let A be an $n \times n$ matrix, then

(a) If A is nonsingular, then A can have a zero column ✗

(b) If A is singular, then the diagonal elements of A are all zero مش شرط

(c) If A is singular, then $adj A$ is also singular ✓

(d) If A is nonsingular and upper triangular, then A^{-1} is lower triangular.

(8) A linear system of three equations in five unknowns has

3x5 underdeter.

(a) infinite number of solutions

(b) no solution

(c) at most one solution

(d) infinite number of solutions or no solution

(9) If A, B are 4×4 nonsingular matrices and $\alpha \in \mathbb{R}$, then

(a) $(A - B)^{-1} = A^{-1} - B^{-1}$

(b) $(\alpha A)^{-1} = \alpha A^{-1}$ ×

(c) $AB \neq 0$

(d) $det(A) = det(B)$

(10) If A, B, C are 4×4 matrices with $det(A) = 2$, $det(B) = -1$, $det(C) = 8$, then $det(-2A^3B^TC^{-1}) =$

$$\frac{-2^4 \; |A|^3 \; |B|}{|C|} = \frac{16 * 8 * -1}{8}$$

(a) 8

(b) 16

(c) -16

(d) -8

(11) If A is a 3×3 matrix such that $A \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, then

$\underset{3\times3}{A}\ \underset{3\times1}{x} = \underset{3\times1}{b}$

a - d
نفس المعنى

(a) A is nonsingular

(b) $det(A) = 0$

(c) The system $Ax = b$ is consistent for every $b \in \mathbb{R}^3$ ×

(d) the system $Ax = 0$ has only one solution

(12) If A, B are $n \times n$ nonsingular matrices, then

(a) $(AB)^T = A^TB^T$ ×

(b) $(A + B)^T = A^T + B^T$ ✓

(c) $(AB)^{-1} = A^{-1}B^{-1}$ ×

(d) $det(\alpha A) = \alpha\, det(A)$ $\alpha^n |A|$

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(Q3) [12 points] Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 3 & 2 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

$$\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$$

(a) Find $det(A)$

$$1\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} + -1\begin{vmatrix} 3 & 0 \\ 0 & 1 \end{vmatrix} + 0\begin{vmatrix} 3 & 2 \\ 0 & 1 \end{vmatrix}$$

$$1(2-0) + -1(3-0) + 0\,(3-0)$$

$$2 + -3 + 0 = -1$$

2

(b) Find $adj A$

حدث العزن.

$$\begin{bmatrix} 1\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} & -1\begin{vmatrix} 3 & 0 \\ 0 & 1 \end{vmatrix} & 0\begin{vmatrix} 3 & 2 \\ 0 & 1 \end{vmatrix} \\ -3\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} & 2\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} & -0\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \\ 0\begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix} & -1\begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} & 1\begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} \end{bmatrix}^T \Rightarrow \begin{bmatrix} 2 & -3 & 0 \\ -3 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}^T = \begin{bmatrix} 2 & -3 & 0 \\ -3 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}_{3\times 3}$$

$$\begin{bmatrix} 2 & -3 & 3 \\ -1 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix}^T$$

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(c) Use $adj A$ to find A^{-1}

$$A^{-1} = \frac{1}{|A|} * adjA$$

$$= -1 * \begin{bmatrix} 2 & -3 & 0 \\ -3 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 3 & 0 \\ 3 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

1

(d) Use Cramer's rule to find the value of x_3 in the system:
$$\begin{aligned} x_1 + x_2 &= 2 \\ 3x_1 + 2x_2 &= 1 \\ x_2 + x_3 &= 0 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 3 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$A_{3\times 3}$

$$X_3 = \frac{|A_3|}{|A|}$$

$$A_3 = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

2

9

$$|A| = 1\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} + -1\begin{vmatrix} 3 & 0 \\ 0 & 1 \end{vmatrix} + 0\begin{vmatrix} 3 & 2 \\ 0 & 1 \end{vmatrix}$$

$$= 2 - 3 = -1$$

$$|A_3| = 1\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} + -1\begin{vmatrix} 3 & 1 \\ 0 & 0 \end{vmatrix} + 2\begin{vmatrix} 3 & 2 \\ 0 & 1 \end{vmatrix}$$

$$= -1 + 0 + 6 = 5$$

$$X_3 = \frac{5}{-1} = -5$$

(Q4) [10 points] If the matrix $\left[\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 0 \\ 2 & 3 & 1 & 1 & -1 \\ 0 & 1 & 1 & a & b \end{array}\right]$ is the augmented matrix of some linear system.

Find the values of a, b that make the system

(i) inconsistent.

if $\alpha = -3$ and $b \neq 1$

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 0 \\ 2 & 3 & 1 & 1 & -1 \\ 0 & 1 & 1 & \alpha & B \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} \boxed{1} & 2 & 1 & -1 & 0 \\ 0 & 1 & 1 & \alpha & b \\ 2 & 3 & 1 & 1 & -1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 0 \\ 0 & \boxed{1} & 1 & \alpha & b \\ 0 & -1 & -1 & 3 & -1 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 0 \\ 0 & 1 & 1 & \alpha & b \\ 0 & 0 & 0 & 3+\alpha & -1+b \end{array}\right]$$

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(ii) has a unique solution.

if $\alpha \neq -3$ and b any value

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 0 \\ 0 & -1 & -1 & 3 & -1 \\ 0 & 1 & 1 & a & b \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 0 \\ 0 & 1 & 1 & -3 & +1 \\ 0 & 0 & 0 & a+3 & b-1 \end{array}\right]$$

0

No values, can make the system has a unique solution $\Rightarrow$ underdetermi: system

$X_3 \Rightarrow$ free variable

(iii) has infinitely many solutions.

if $\alpha = -3$ and $b = 1$

or $a \neq -3$ & $b \in \mathbb{R}$

or $a \neq -3$, b any number

2

under. system

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Q5) [10 points] An $n \times n$ matrix A is called idempotent if $A^2 = A$.

(a) Show that the determinant of an idempotent matrix is either zero or one.

idempotent $\Rightarrow A^2 = A$

take determinants for both side

$\det A^2 = \det A$

$(\det A)^2 = \det A$ [$\det A$ is constant]

$-\det A \quad -\det A$

$(\det A)^2 - \det A = 0$

عامل مشترك $\det A (\det A - 1) = 0$

$\det A = 0$ or $\det A - 1 = 0$

$\det A = 1$

(b) Show that if A is a nonsingular idempotent matrix, then $A^{-1} = adj A$

$A \operatorname{adj} A = |A| I$

~~$A^{-1} = \frac{1}{|A|} \operatorname{adj} A$~~

A^{-1} is exist [nonsingular]

$A^{-1} A \operatorname{adj} A = A^{-1} |A| I$

$\operatorname{adj} A = A^{-1} |A|$

from the previous proof if A is idempotent $\left[\begin{array}{ll} |A| = 0 & \text{if } A \text{ is singular} \\ |A| = 1 & \text{if } A \text{ is nonsingular} \end{array} \right]$

from the question A is nonsingular so $\Rightarrow |A| = 1$

$\operatorname{adj} A = A^{-1} * 1$

$\operatorname{adj} A = A^{-1}$

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