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Birzeit University
Mathematics Department
Math 234-1

First Exam

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Q1: (70 points): This part consists of seven questions, each question has five answers. Mark each answer of each question by true or false.

(1) Let A be 3×3 matrix such that $|A| = 5$. Then.

T (a) $|adj(A)| = 25$ True $|adj(A)| = |A|^{n-1}$

T (b) $adj(A)$ is nonsingular

F (c) $|A^{-1}| = \frac{1}{25}$ $|A|^{n-2} A$

T (d) A is nonsingular

T (e) $adj(adj(A)) = 5A$

(2) Let A, B be $n \times n$ matrices such that $AB = 0$. Then.

F (a) A or B is a zero matrix

F (b) A and B are singular matrices

F (c) $|A| = 0$ and $|B| = 0$

T (d) $|AB| = 0$

F (e) At least A or B is a nonsingular matrix x

(3) Let A be 3×3 nonzero matrix such that $a_1 = 3a_3$ and $a_1 - a_2 + 3a_3 = 0$. Then.

F (a) $Ax = 0$ has a unique solution $2a_1 - a_2 = 0$ $(1, -1, 3)$

T (b) $Ax = 0$ has infinite solutions $a(2, 1, 0)$

T (c) A is singular

F (d) A is nonsingular

F (e) The solutions of $Ax = 0$ are of the form $a(1, 0, -2)^t + b(1, -1, 3)^t$, where $a, b \in R$

(4) Let A, B be $n \times n$ row equivalent matrices. Then.

F (a) $|A| = |B|$

F (b) $|A| = |B^t|$

T (c) A is nonsingular iff B is nonsingular

T (d) A is nonsingular iff A^t is nonsingular

T (e) $|A| = |A^t|$

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(5) Let A be 3×3 matrix such that $|A| = 5$. Then.

F (a) $|5A| = 25$

T (b) $Ax = 0$ has only the zero solution

F (c) A is singular

T (d) A is nonsingular

T (e) $Ax = b$ is consistent for every $b \in R^3$

$5^n \, |A| =$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 2 & 0 \end{bmatrix}$$

(6) Let A be $n \times n$ matrix. Then.

F (a) A always has an LU factorization

F (b) If A has an LU–factorization then A is nonsingular iff L is nonsingular

T (c) $|A| = |U|$

T (d) If A has an LU–factorization then A is nonsingular iff U is nonsingular

T (e) If A has an LU–factorization then A is row equivalent to U

(7) Let A, B be 3×3 matrices , $|A| = 6, |B| = 3$. Then.

T (a) $|2A^{-1}B| = 4$

F (b) $|2A^t B| = 36$

T (c) $|adj(AB)| = 324$

F (d) $|A + B| = 9$

F (e) A or B is singular

$\frac{2^3}{6} \cdot 3 = 4$

$8 \times 6 \times 3$

$|adj(AB)| = |AB|^{n-1}$

$= (|A|\,|B|)^{n-1}$

$$\begin{array}{r} 6 \\ 18 \\ \hline 018 \\ 144 \\ 180 \\ \hline 324 \end{array}$$

Q2 (10 points) Let $A = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ -1 & 1 & 1 & 4 \\ 1 & 2 & \alpha & \beta \end{array}\right)$ be the Augmented matrix of a linear system.

Find the values of α, β so that

(a) the system is consistent

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 2 & 2 & 6 \\ 0 & 1 & \alpha - 1 & \beta - 2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 1 & \alpha - 1 & \beta - 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & \alpha - 2 & \beta - 5 \end{array}\right]$$

case 1) $\boxed{\alpha = 2 \ \& \ \beta = 5}$

Case 2) $\boxed{\boxed{\alpha \neq -2} \ \& \ \beta \text{ is any number in } \mathbb{R}}$

Case 3) $\boxed{\beta = 5 \ \& \ \alpha \text{ is any number in } \mathbb{R}}$

Case 1 $\in$ Case 3

(b) the system is inconsistent

$\boxed{\alpha = 2 \ \& \ \beta \neq 5}$

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Q3 (15 points) Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$

(a) Find the LU-factorization of A

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix} \cong \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 2 & -1 \end{bmatrix} \cong \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -3 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -3 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

(b) Find the inverse of A

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 3 & -1 & 0 & 0 & 1 \end{array}\right] \cong \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -3 & -1 & -2 & 1 \end{array}\right] \cong \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & +\frac{1}{3} & +\frac{2}{3} & -\frac{1}{3} \end{array}\right]$$

(c) Solve the homogeneous system whose coefficient matrix is A above

A is non singular, then the system has the trivial solution only Soln $= (0, 0, 0)$

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$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{4}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 1 & 0 & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \end{array}\right] \Rightarrow A^{-1} = \frac{1}{3}\begin{bmatrix} 4 & -1 & -1 \\ -1 & 1 & 1 \\ 1 & 2 & -1 \end{bmatrix}$$

as

Q4: (10 points)

Let A be $n \times n, n \geq 2$ matrix.

(a) Write the formulae that gives the relation between A and $adj(A)$

$$A \cdot adj(A) = |A| \cdot I$$

(b) Show that $|adj(A)| = |A|^{n-1}$

$$|A \cdot adj(A)| = ||A| \cdot I|$$

$$|A| \cdot |adj A| = |A|^n \, |I| -$$

$$|adj.(A)| = |A|^{n-1}$$

Assigns

(c) If A is nonsingular. Show that $adj(adj(A)) = |A|^{n-2} A$

$$A \cdot adj(A) = |A| \cdot I$$

$$(adj A)^{-1}(adj A \cdot adj\,adj(A)) = (|adj A| \; I)(adj A)^{-1}$$

$$adj(adj(A)) = |adj A| (adj A)^{-1} \quad \ldots (1)$$

$$(A \cdot adj A)^{-1} = (|A| \; I)^{-1}$$

$$(adj A)^{-1} A^{-1} = |A^{-1}| \, I$$

$$(adj A)^{-1} = \frac{A}{|A|}$$

$\therefore$ subs in (1)

$$|adj A| = |A|^{n-1}$$

$$adj(adj(A)) = \frac{|A|^{n-1} A}{|A|} = |A|^{n-2} A$$

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Q5: **Bonus(10 points)** Let A be $n \times n, n \geq 2$ matrix. Prove that A is nonsingular iff $adj(A)$ is nonsingular

$A \cdot adj A = |A| I$

$|A| \, |adj A| = |A|^n$

$|adj A| = |A^{\cancel{n-1}}|^{n-1}$

then A most be non singular when adj A is non singular

if A is singular $|A|^{n-1} = 0$

so $|adj A| = 0$ then adj A is singular

by contrast $|A|^{n-1} \neq 0$ A is non singular

then $|adj A| \neq 0$ so adj A is non singular

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