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$\frac{94+2}{100}$

Birzeit University
Department of Mathematics
Math 243

Second hour exam
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Section#...12830 – 14800 ...

Spring 2014
Number:..1121649

Questio#1(30%) Prove or disprove each of the following statements

a)If $A \neq \varnothing$ and $A\times B = A\times C$ then B=C   True

Proof suppose $A\times B = A\times C$ and

$\boxed{\subseteq}$ let $y \in B \Rightarrow (x,y) \in A\times B$ (since $A \neq \varnothing$)

$\Rightarrow (x,y) \in A\times C$ (since $A\times B = A\times C$)

$\Rightarrow y \in C$

$\Rightarrow B \subseteq C$

$\boxed{\supseteq}$ let $y \in C \Rightarrow (x,y) \in A\times C$ (since $A \neq \varnothing$)

$\Rightarrow (x,y) \in A\times B$ (since $A\times C = A\times B$)

$\Rightarrow y \in B \Rightarrow C \subseteq B \Rightarrow B = C$

b)If f and g are functions then $f \cup g$ is a function   False

counterexample

$f = \{(1,2)\} \rightarrow$ function
$g = \{(1,3)\} \rightarrow$ function
$f \cup g = \{(1,2),(1,3)\}$ not function

c)If R and S are equivalence relations on A then $R \cup S$ is an equivalence relations on A

False
not transitive

Counter example

$R = \{(4,6), (2,3)\}$ tran.
$S = \{(1,2), (3,4)\}$ tran.
$R\cup S = \{(4,6), (2,3), (1,2), (3,4)\}$
not tran. Since $(2,4) \notin R\cup S$

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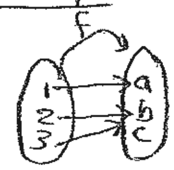
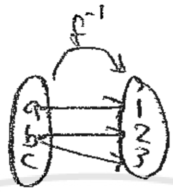
d)If f is a function then f^{-1} is a function   False

counter example



function



not function

e) If R,S are transitive then $S \circ R$ is transitive

False

Counterexample

$R = \{(1,2),(3,4)\}$ tran.

$S = \{(4,6),(2,3)\}$ tran.

$S \circ R = \{(1,3),(3,6)\}$ not transitive since $(1,6) \notin S \circ R$

f) If R and S are symmetric relations then $(R \circ S)^{-1} = S \circ R$

~~False~~ ~~counter example~~

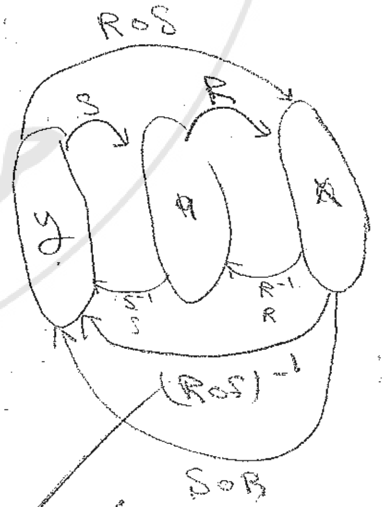
True

proof

$(x,y) \in (R \circ S)^{-1} \Leftrightarrow \exists a,\ (x,a) \in R^{-1}$ and $(a,y) \in S^{-1}$

$\Leftrightarrow (x,a) \in R$ and $(a,y) \in S$

$\Leftrightarrow (x,y) \in S \circ R$



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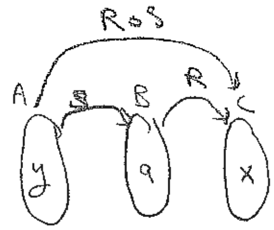
Question #2(20%) a)Prove that if A,B,C be sets, and let $R \subseteq B\times C, S \subseteq A\times B$ be relations then $(R\circ S)^{-1} = S^{-1}\circ R^{-1}$

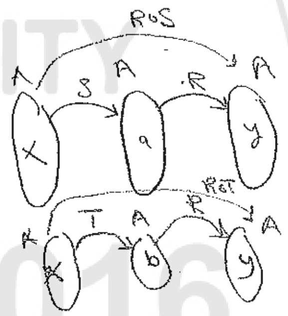


proof $\quad (x,y) \in (R\circ S)^{-1} \Leftrightarrow (y,x) \in R\circ S$

$\Leftrightarrow \exists a; (y,a) \in S$ and $(a,x) \in R$

$\Leftrightarrow \exists a \in A ; (a,y) \in S^{-1}$ and $(x,a) \in R^{-1}$

$\Leftrightarrow (x,y) \in S^{-1}\circ R^{-1}$



b)Prove that if R,S,T be relations from A to A then $(R\circ S)\cup(R\circ T)\subseteq R\circ(S\cup T)$

$R,S,T \subseteq A\times A$

proof

let $(x,y) \in (R\circ S)\cup(R\circ T) \Rightarrow (x,y)\in R\circ S$ or $(x,y) \in R\circ T$

$\Rightarrow \exists a ; (x,a) \in S$ and $(a,y) \in R$

or $\exists b ; (x,b) \in T$ and $(b,y) \in R$

~~⇒ (x,a) ∈ S and (x,b) ∈ T or (x,a) ∈ S and (b,y) ∈ R or (a,y) ∈ R and (x,b) ∈ T or (a,y) ∈ R and (b,y) ∈ R~~

$\Rightarrow ((x,b) \in S$ or $(x,b) \in T)$ and $(b,y) \in R$

$\Rightarrow (x,b) \in (S\cup T)$ and $(b,y) \in R$

$\Rightarrow (x,y) \in R\circ(S\cup T)$

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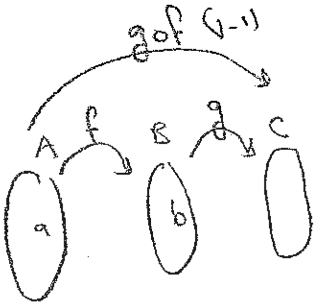
Question #3(20%) Let A, B, C be nonempty set and let $f: A \to B$, $g: B \to C$ be functions.

a) Show that if $g \circ f$ is one to one, then $f: A \to B$ is one to one.



f (1-1)??

proof suppose $f(a) = f(b)$

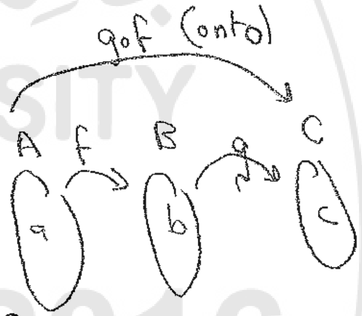
$\Rightarrow g(f(a)) = g(f(b))$

$\Rightarrow g \circ f(a) = g \circ f(b)$

$\Rightarrow$ ~~f(a)~~ $a = b$ Since $g \circ f$ is (1-1)

$\Rightarrow f$ is one to one

b) Show that if $g \circ f$ is onto, then $g: B \to C$ is onto.



g onto??

proof let $c \in C \Rightarrow \exists a \in A ; g \circ f(a) = c$ (onto)

$\Rightarrow (a, c) \in g \circ f$

$\Rightarrow \exists b \in B ; (a, b) \in f$ and $(b, c) \in g$

$\Rightarrow \exists b \in B ; (b, c) \in g$

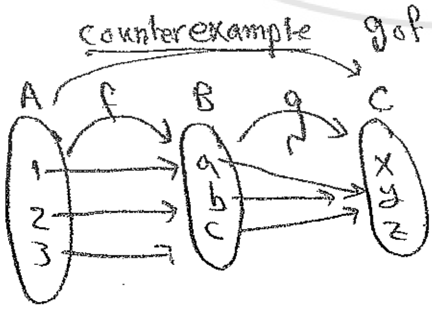
$\Rightarrow g(b) = c$

g is onto

c) Show that the converse of (a) is not true

converse : If $f: A \to B$ is one to one, then $g \circ f$ is one to one.

False counterexample



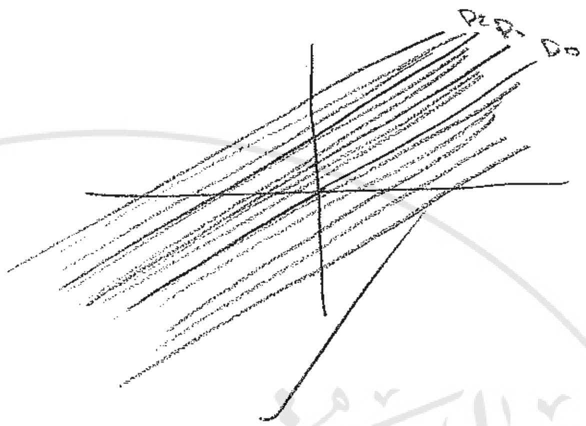
f is one to one but gof is not one to one

Question #4(20%)

Let X=$\mathbb{R}\times\mathbb{R}$, For each real number b let $D_b = \{(x, y)\in X:\ y= x+b\}$

a) Is $\{D_b :\ b\in\mathbb{R}\}$ a partion of X? Prove your answer?

$D_1 = \{(x,y)\in X : y = x+1\}$



$D_0 =\ \ =\ \ y = x$

$D_{-1} =\ \ "\ \ y = x-1$

⋮

هنا الرسم

[1] $\phi \notin D_b$

[2] لا يوجد تقاطع بين خط و خط

since they are parallel and the slope = 1 for all $b\in\mathbb{R}$

[3] $\bigcup_{b\in R} D_B = \mathbb{R}\times\mathbb{R}$ اتحاد الخطوط كلها يعطي $\mathbb{R}\times\mathbb{R}$

$\Longrightarrow D_b$ is apartion of X

b)Define a relation R on X by (s,t)R(u,v) if and only if there is a real number b such that (s,t)and (u,v) both belongs to D_b, for some $b\in\mathbb{R}$

Is R Reflexive ? Symmetric? Transitive? Explane your answer

$(s,t)\ R\ (u,v)$ iff $t-s = b = v-u \Rightarrow t-s = v-u$

ref $(x,y)\ R\ (x,y)$ since $y-x = y-x$ ✓ ref ✓

symm $(a,b)\ R\ (c,d) \Rightarrow b-a = d-c$

$\Rightarrow d-c = b-a$

$\Rightarrow (c,d)\ R\ (a,b)$ ✓ symm ✓

tran let $(a,b)\ R\ (c,d)$ and $(c,d)\ R\ (e,f)$

$\Rightarrow b-a = d-c$ and $d-c = f-e$

$\Rightarrow b-a = f-e$

$\Rightarrow (a,b)\ R\ (e,f)$ ✓ tran. ✓

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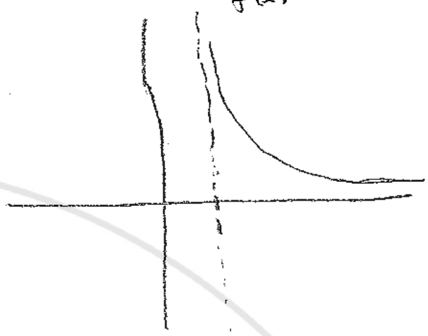
Question #5(10%) a) Let $f(x)=\frac{1}{\sqrt{x-1}}$, $g(x)=\sqrt{4-x}$. Find the domain of f , g , f ∘ g

* $f(x)=\frac{1}{\sqrt{x-1}}$

domain $x-1>0 \Rightarrow x>1$

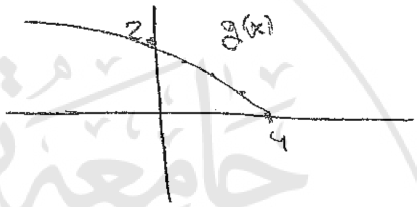
range $(0,\infty)$



* $g(x)=\sqrt{4-x}$

domain $4-x \geq 0 \Rightarrow x \leq 4$

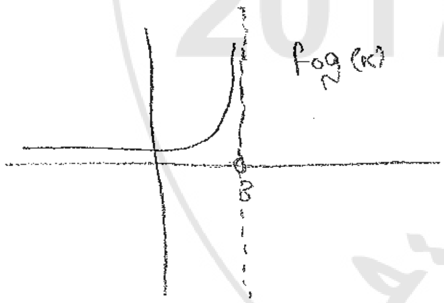
range $[0,\infty)$



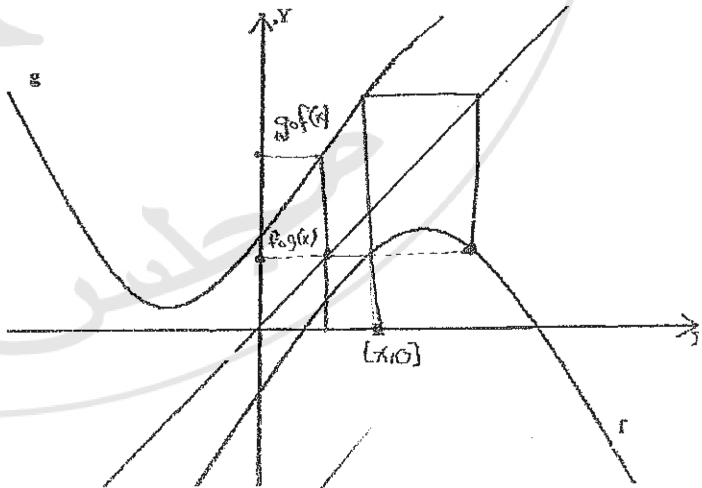
* $f\circ g = f(g(x)) = f(\sqrt{4-x}) = \frac{1}{\sqrt{\sqrt{4-x}-1}}$

domain $\sqrt{4-x}-1>0 \Rightarrow \sqrt{4-x}>1 \Rightarrow 4-x>1 \Rightarrow x<3$

range $(0,\infty)$



b)Use the graph of the functions f , and g
to indicate on the graph
the value of (f ∘ g) (x) and (g ∘ f)(x)



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