Birzeit University- Mathematics Department Math234, Linear Algebra

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Qı	estion 1(24%) Answer by True or False:	
	1. It is always true that $det(AB) = det(BA)$ for any $n \times a$	n matrices A, B. ()
	2. If A and B are nonsingular $n \times n$ matrices that s $A^{-1}B^{-1} = B^{-1}A^{-1}$.	/
,	3. If $AB = O$ then either $A = O$ or $B = O$.	(.**)
4 det()	14.) There is a nonzero 2×2 matrix such that $\det(2A) = 5$. The matrix AA^T is symmetric. $(AA^T)^T = A^TA$.	2 det(A). V (X.) (X.)
) SUBlay		ays consistent . (X)
	7. If \vec{x} is a solution to $A\vec{z} = \vec{b}$ and \vec{y} is a solution to solution to the system $A\vec{z} = \vec{b}$. $A(\vec{x}+\vec{y}) = A\vec{x} + A\vec{y}$	y = b.
	8. If A and B are nonsingular $n \times n$ matrices then $A +$	B is nonsingular. (X)
	9. The product of two elementary matrices is an elementary	ntary matrix. (X)
AR+RA	10. $(A+B)(A-B) = A^2 - B^2$.	(X)
	11. If A is $n \times n$ matrix in which $r_i = cr_j$ then $\det(A) =$	= 0. (.v)
	12. The transpose of an elementary matrix is an elementary type.	entary matrix of the same
1	13. All singular matrices are row equivalent.	(X)
	14. If A is nonsingular then $adj(A)$ is nonsingular.	(.v)
600	15. $\det(A^k) = (\det(A))^k.$	()
	16. We can use Cramer's rule to solve any linear system. Just for non-Singular Matrices.	m. (.**)

Question 2(30%) Circle the correct answer (Show your work):

- 1. Consider the homogeneous system $A\vec{x} = \vec{0}$ then
 - (a) If \vec{x} is a solution then $c\vec{x}$ is a solution for any constant c.
 - (b) If \vec{x} and \vec{y} are two solutions then $\vec{x} + \vec{y}$ is also a solution.
 - The system is always consistent.
 - (d) All of the above.
- 2. If $\vec{b} = \vec{a}_1 + \vec{a}_2 = \vec{a}_3 + \vec{a}_4$ then the 3×4 system $A\vec{x} = \vec{b}$ is

 (a) Inconsistent.

 (b) Was only one solution

 - Has infinitely many solutions.
 - (d) None of the above.
- 3. If A is a nonsingular matrix then
 - (a) The reduced row echelon form of A is the identity matrix.
 - (b) A is the product of elementary matrices.
 - (c) $det(A) \neq 0$.
 - All of the above.

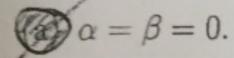
Consider the matrix

$$A = \begin{pmatrix} 1 & \alpha & \beta & 0 \\ 0 & \alpha & \beta & 1 \\ 0 & 0 & \alpha & \beta \end{pmatrix} = 0$$

Use this matrix to answer questions (4,5,6)

- 4. The matrix is in row echelon form if
 - (a) $\alpha = 0$.

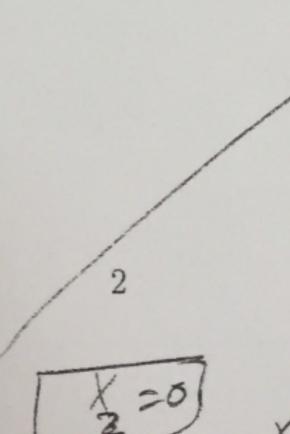
 - (d) $\beta = 1$.
- 5. The matrix is in reduced row echelon form if

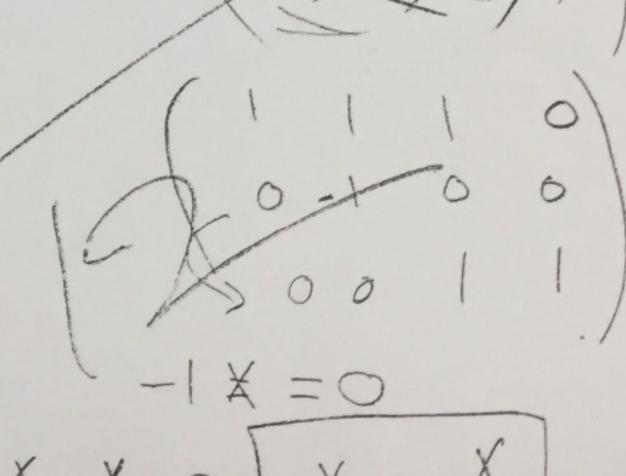


(b)
$$\alpha = \beta = 1$$
.

(c)
$$\alpha = 1, \beta = 0.$$

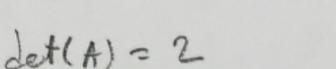
(d)
$$\alpha = 0, \beta = 1$$
.





- 6. If $\alpha = \beta = 1$, the set of all solutions of the system $A\vec{x} = \vec{0}$ is
 - (a) $(a, -a, a, a)^T, a \in \mathbb{R}$.
 - (b) $(a, 0, a, a)^T, a \in \mathbb{R}$.
 - $(a,0,-a,a)^T$, $a \in \mathbb{R}$.
 - (d) $(a, 0, a, -a)^T, a \in \mathbb{R}$.
- 7. If \vec{x}_1 and \vec{x}_2 are two solutions of the system $A\vec{x} = \vec{b}$ then $c_1\vec{x}_1 + c_2\vec{x}_2$ is also a
 - (a) $c_1 = c_2 = 0$.

 - (c) $c_1 + c_2 = 0$.
 - (d) $c_1 = c_2 = 1$.
- /8. If A is a 3×3 matrix with det(A) = 2 then det(2adj(A)) =



(a) 8.

- 9. One of the following statements is false:
 - (a) If det(A) = 1 then $A^{-1} = adj(A)$.
 - A(adj(A)) = I.
 - (c) If A is singular then det(A) = 0.
 - (d) If A is singular then the system $A\vec{x} = \vec{0}$ has nontrivial solution.
- 10. The triangular factorization of the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ is

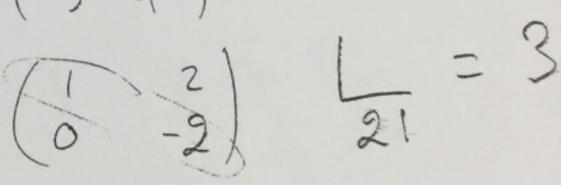
(a)
$$L = \begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix}, U = \begin{pmatrix} 1 & 2 \\ 0 & -2 \end{pmatrix}$$

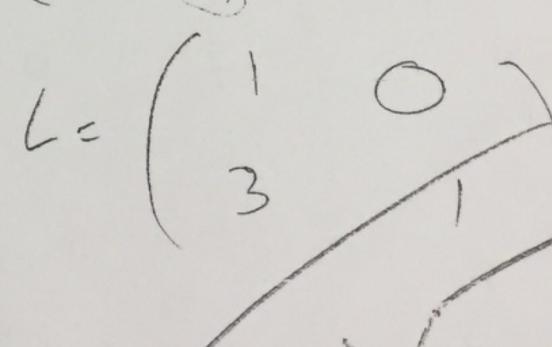
$$L = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}, U = \begin{pmatrix} 1 & 2 \\ 0 & -2 \end{pmatrix}$$

$$(c) L = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}, U = \begin{pmatrix} 1 & 2 \\ 0 & 4 \end{pmatrix}$$

(c)
$$L = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}, U = \begin{pmatrix} 1 & 2 \\ 0 & 4 \end{pmatrix}$$

(d)
$$L = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}, U = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$$





Question 3(15%) Find the inverse of the following matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 7 & 9 \end{pmatrix}$$

$$13 - 16 + 6 = 3$$

$$26 - 40 + 14 = 40 - 40 = 0$$

$$39 - 86 + 18 = 3$$

$$26 - 40 + 18 = 40 - 40 = 0$$

$$39 - 86 + 18 = 3$$

Question 3(15%) Find the inverse of the following matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 7 & 9 \end{pmatrix}$$

$$39 - 86 + 18 = 96$$

$$39 - 18 = 96$$

Question 4(15%) A matrix A is called orthogonal if $AA^T = A^TA = I$.

(a) Show that type I elementary matrices are orthogonal. Hint: use the fact that

type I is symmetric
$$\Rightarrow E^T = E$$
.

 $E^T = E = E$

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AND: $E^T = E = E = E = E$

AND: $E^T = E = E = E = E$

(b) Show that the product of two orthogonal matrices is orthogonal.

A is orth. How
$$AA^{T} = A^{T}A = I$$
, B is orth. And $BB = BB = I$

$$A^{T} = A^{T} = (A^{T})^{T}$$

$$B^{T} = B^{T} = (B^{T})^{T}$$

$$(AB)(AB)^{T} = AB(B^{T}A^{T}) = AB(B^{T}A^{T}) = AB(AB^{T}) = I$$

$$(AB)^{T}(AB) = (B^{T}A^{T})(AB) = (B^{T}A^{T})(AB) = (AB)^{T}(AB)$$

$$(C) \text{ Show that if } A \text{ is orthogonal then } \det(A) = \pm I.$$

$$-A^{T}A = 1$$
 if we take the determinate
$$\det(A^{T}A) = \det(\overline{1})$$
 but
$$\det(A^{T}) = \det(A)$$

$$\left(\det(A)\right)^{2} = 1$$
 "
$$\det(A^{T}) = \left(\det(A)\right)^{3}$$
if we take the determinate
$$\det(A^{T}A) = \det(A^{T}) = \det(A)$$

Question 5(16%) Use Gauss-Jordan reduction to solve the following linear system $\alpha x_1 + x_2 + x_3 = 1$ $x_1 + \alpha x_2 + x_3 = 1$ We have infinitely many $x_1 + x_2 + \alpha x_3 = 1$ Consider the following cases: (a) $\alpha = 1$. but because we have floofree Varible and reduce tow exhaux

(b) $\alpha = 0$. r3=13-11.

