Birzeit University
Mathematics Department
Math 234

First Exam-answers

Student Name: .... Number:..................
Sections:
Mohammad Saleh .... (1 at 11), ... (4 at 12:20)

Q1 (40 points) Answer the following statements by true or false:

(1) If A, B are square $n \times n$ nonzero matrices such that $AB = 0$, then A and B are singular. T

(2) If $A = LU$ is the LU-factorizaton and A is singular then U is singular. T

(3) If A is symmetric and skew symmetric then A must be a zero matrix.(A is skew symmetric if $A^T = -A$). T

(4) If A is an $n \times n$ nonsingular matrix then $det(adj(A)) = (det(A))^{n-1}$. T

(5) If the system $Ax = b$ is consistent then b is a linear combinations of the columns of A. T

(6) If A, B are square $n \times n$ matrices and AB is singular then A and B are singular. F

(7) If A is row equivalent to B then $\det(A) = \det(B)$. F

(8) If the coefficient matrix of the system $AX = 0$ is singular then the system has infinitely many solutions. T

(9) In the linear system $Ax = b$, if b is a linear combinations of the columns of A then the system has a unique solution. F

(10) If the row echelon form of the matrix A involves a free variable, then the linear system $Ax = b$ has infinitely many solutions. F

(11) a square matrix A is nonsingular iff its row echelon form is the identity matrix. F

(12) If $AB = AC$, $A \neq 0$, then $C = B$. F

(13) In the linear system $Ax = b$, if b is the first column of A, then the system has infinitely many solutions. F

(14) If $det(A) = det(B)$, then $A = B$. F

(15) A square matrix A is nonsingular iff its RREF is the identity matrix. T

(16) In the linear system $AX = b$, if $b = a_1 - a_2 + 3a_4, a_1 = -a_3$ then the system has infinite solutions. T

(17) If the coefficient matrix of the system $Ax = 0$ is singular then the system has infinite solutions. T

(18) The vector $(0, 0, 0)^T$ is a linear combination of the vectors $(1, 2, 3)^T, (1, 4, 1)^T, (2, 3, 1)^T$. T

(19) If $A \neq B$, then $det(A) \neq det(B)$. F

(20) If A and B are not invertible, then $det(A) = det(B)$. T

(21) If an $n \times n$ matrix A is row equivalent to I_n then $\det(A) = \pm 1$. F

Q2:(45points) Circle the correct answer:

1 If A is a 4×3 matrix such that $Ax = 0$ has only the zero solution, and $b = \begin{pmatrix} 1 \\ 3 \\ 2 \\ 0 \end{pmatrix}$, then the system $Ax = b$

(a) has exactly one solution

(b) is either inconsistent or has a unique solution. T

(c) is either inconsistent or has an infinite number of solutions

(d) is inconsistent

(2) Let A be a 3×3 matrix such that $Ax = 0$ for a nonzero x, then

(a) $|A| \neq 0$

(b) A is nonsingular

(c) A is row equivalent to the identity

(d) $|A| = 0$. T

(3) If E is an elementary matrix, then one of the following statements is not true

(a) $E + E^T$ is an elementary matrix. T

(b) E^{-1} is an elementary matrix.

(c) E^T is an elementary matrix.

(d) E is nonsingular.

(4) If A and B are $n \times n$ matrices such that $Ax = Bx$ for some none zero $x \in R^n$. Then

(a) $A - B$ is singular. T

(b) $A - B$ is nonsingular.

(c) $A = B$

(d) None.

(5) If A is a 3×3 matrix with $\det(A) = -2$. Then $\det(adj(A)) =$

(a) -2.

(b) 4. T

(c) -4.

(d) -8.

(d) None.

(6) If $AB = 0$, where A and B are $n \times n$ matrices. Then

(a) either $A = 0$ or $B = 0$

(b) both A, B are singular.

(c) both A, B are nonsingular.

(d) either A or B is singular. T

(7) If B is a 3×3 matrix such that $B^2 = B$. Then

(a) B is nonsingular.

(b) $\det(B) = 0$.

(c) $B^5 = B$. T

(d) $B = I$.

(8) The adjoint of the matrix $\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$ is

(a) $\begin{pmatrix} 3 & 2 \\ -1 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix}$.

(c) $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}$ T

(e) None

(9) An $n \times n$ matrix A is invertible if

(a) there exists a matrix B such that $AB = I$. T

(b) $|A| = 0$

(c) $Ax = 0$ has a nonzero solution

(d) All of the above

(10) Let A be nonsingular. Then

(a) If A is symmetric then A^{-1} is symmetric

(b) If A is diagonal then A^{-1} is diagonal

(c) Both (a) and (b) T

(d) None of the above

(11) The LU decomposition of the matrix $\begin{bmatrix} 2 & 4 & 2 \\ 1 & 5 & 2 \\ 4 & -1 & 9 \end{bmatrix}$ is

(a) $L = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ -2 & -3 & 1 \end{bmatrix}, U = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 8 \end{bmatrix}$

(b) $L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 2 & -3 & 1 \end{bmatrix}, U = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 8 \end{bmatrix}$. T

(c) $L = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix}, U = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 8 \end{bmatrix}$

(d) None

(12) If $A^2 = I$ then

(a) $A - I$ and $A + I$ both cannot be nonsingular. T

(b) $A - I$ and $A + I$ are nonsingular.

(c) $A - I$ and $A + I$ are singular.

(d) $(A - I)^{-1} = A + I$

(13) Let A, B be 3×3 matrix $|A| = |2B| = 4$. Then $\det((2AB)^{-1}) =$

(a) 4.

(b) 16.

(c) 1.

(d) None. T

(14) Assume that the last row in the row echelon form of a 4×4 linear system is $[0 \quad 0 \quad 0 \quad a - 3 | b - 4]$. The system has one solution if

(a) $b \neq 4$.

(b) $a \neq 3$. T

(c) $a \neq 3$ and $b \neq 4$.

(d) $a = 3$, $b = 4$.

(15) Let A be a 4×4 matrix. If the homogeneous system $Ax = 0$ has only the trivial solution then

(a) A is nonsingular.

(b) A is row equivalent to I.

(c) RREF of A is I.

(d) All of the above. T

Q3: (10 points) Let A, B be $n \times n$ nonzero matrices such that $AB = 0$.

1. Show that A must be singular
   Suppose not, that is A is nonsingular, multiply from left $AB = 0$ by A^{-1}, we get $B = 0$, a contradiction.
2. Show that $Ax = 0$ has infinitely many solutions
   Since A is singular, so $Ax = 0$ has infinite solutions

Q4: (10 points)

1. Let A be $n \times n, n \geq 2$. Show that if A is nonsingular, then $adj(A)$ is nonsingular
   $Aadj(A) = |A|I$. Since A is nonsingular, so A^{-1} exists, and so $adj(A) = |A|A^{-1}$ which is nonsingular since $|A| \neq 0$. Or, you can use the determinant: $|A| \neq 0$, so $|adj(A)| = |A|^{n-1} \neq 0$, and so $adj(A)$ is nonsingular.
2. Let A, B, AB be $n \times n$ square symmetric matrices. Show that $AB = BA$
   $AB = (AB)^T = B^T A^T = BA$

Q5(10 points)

(a) Use Gauss elimination method to solve the linear system whose augmented matrix $\begin{pmatrix} 1 & 1 & 1 & | & 2 \\ -1 & 1 & 1 & | & 4 \end{pmatrix}$ Solution $(-1, 3-\alpha, \alpha)^T, \alpha \in R$

(b) Find the inverse of $\mathbf{A} = \begin{pmatrix} 1 & 1 & 0|1 & 0 & 0 \\ -1 & 1 & 1|0 & 1 & 0 \\ 1 & 3 & -1|0 & 0 & 1 \end{pmatrix}$ $R_2{+}R_1, R_3{-}R_1 \rightarrow \begin{pmatrix} 1 & 1 & 0|1 & 0 & 0 \\ 0 & 2 & 1|1 & 1 & 0 \\ 0 & 2 & -1|-1 & 0 & 1 \end{pmatrix}$

$R_3 - R_2 \rightarrow \begin{pmatrix} 1 & 1 & 0|1 & 0 & 0 \\ 0 & 2 & 1|1 & 1 & 0 \\ 0 & 0 & -2|-2 & -1 & 1 \end{pmatrix}$ $R_2 + \frac{1}{2}R_3 \rightarrow \begin{pmatrix} 1 & 1 & 0|1 & 0 & 0 \\ 0 & 2 & 0|0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & -2|-2 & -1 & 1 \end{pmatrix}$

$\frac{1}{2}R_2, -\frac{1}{2}R_3 \rightarrow \begin{pmatrix} 1 & 1 & 0|1 & 0 & 0 \\ 0 & 1 & 0|0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 1|1 & \frac{1}{2} & \frac{-1}{2} \end{pmatrix}$ $R_1 - R_2 \rightarrow \begin{pmatrix} 1 & 0 & 0|1 & \frac{-1}{4} & \frac{-1}{4} \\ 0 & 1 & 0|0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 1|1 & \frac{1}{2} & \frac{-1}{2} \end{pmatrix}$.

So $A^{-1} = \begin{pmatrix} 1 & \frac{-1}{4} & \frac{-1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} \\ 1 & \frac{1}{2} & \frac{-1}{2} \end{pmatrix}$

Or, you can use the cofactor method (not preferable)