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First Hour-Exam **Summer Semester 2011/2012**

Student Name: [redacted] **Number:** [redacted] **Section** 8

Q1 (24 points) Answer the following statements by true or false:

T (1) If A is an invertible matrix then A^t is invertible. T

F (2) A nonhomogeneous system which has a none zero solution must have infinitely many solutions. F

F (3) If the square system $AX = 0$ has a nonzero solution then A is singular. F

T (4) If A, B are two square matrices and AB is invertible then both A and B are invertible. T

T (5) If A, B are two square none zero matrices and $AB = 0$ then both A and B are not invertible. T

$A \neq 0$, $B \neq 0$ $\rightarrow |A| = 0 \rightarrow$ sing.

F (6) If $(A|b)$ is the augmented matrix of the system $AX = b$ where b is any column of A then the system is consistent F

T (7) If the square system $AX = 0$ has only the zero solution then $A^n X = 0$ has only the zero solution for every positive integer n. T

T (8) If A is symmetric (A is symmetric means $A^t = A$) and skew symmetric (A is skew-symmetric means $A^t = -A$) then A must be zero. T

$A = A^T$

F (9) Let A be a square 3×3 matrix. Then $|3A| = 9|A|$. F

$3^3 |A| = 27 |A|$

F (10) Let A be a square $n \times n$ nonsingular matrix. Then $|adj A| = |A|^{n-2}$. F

$|A^{-1}|A||$

T (11) A homogeneous system is always consistent T

F (12) If A,B are $n \times n$ matrices and $AB = 0$. Then $(A + B)^2 = A^2 + BA + B^2$. T

T (13) If $(A|b) = \left(\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 1 & 1 & 2 & 1 \end{array}\right)$ is the augmented matrix of the system $AX = b$ then the system has no solution T

$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 1 & 1 & 2 & 1 \end{array}\right) \xrightarrow{-R_1 + R_2} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & 0 & 0 & -3 \end{array}\right)$

F (14) If the number of unknowns in a linear system exceeds the number of equations then the system has infinitely many solutions.

$m < n$

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$|A^{-1}|A|| = |A|^{-n} A$ 20 $|adj| = |A^{-1}|A||$ $(A+B)(A+B)$

$|A|^{-n} A$

$A^2 + BA + AB + B^2$

T (15) If matrix is in reduced echelon form the it is in echelon form.

$m < n$

F (16) If the number of unknowns in a linear system exceeds the number of equations then the system has infinitely many solutions.

$m < n$

T (17 If the number of unknowns in a homogeneous linear system exceeds the number of equations then the system has infinitely many solutions.

T (18) The product of two elementary matrices of the same size is invertible.

F (19) The product of two elementary matrices of the same size is elementary.

T (20) The product of two singular matrices of the same size is singular.

F (21) The sum of two singular matrices of the same size is singular.

F (22) Row equivalent matrices have the same determinant.

$|A| \neq 0$

T (23) A diagonal matrix is invertible iff all entries in the main diagonal are nonzeros

F (24) If the linear system $AX = b$ has a unique solution then the system $Ax = c$ has a unique solution.

Q2 :(11 points) Circle the most correct answer

(1) Let A be nonsingular. Then

(a) If A is symmetric then A^{-1} is symmetric

(b) If A is triangular then A^{-1} is triangular

(c) If A is diagonal then A^{-1} is diagonal

(d) All of the above

(2) If A is a 4×4 matrix such that A is singular, and $b = \begin{pmatrix} 0 \\ 3 \\ 2 \\ 1 \end{pmatrix}$, then

(a) It is possible that $AX = b$ has infinitely many solutions

(b) The system $AX = b$ has exactly one solution

(c) The system $AX = b$ is always inconsistent

(d) The system is always consistent

(3) If the coefficient matrix of the linear system $AX = b$ is a 3×4 then

(a) The system is consistent

(b) The system is inconsistent

(c) The system has a unique solution

(d) None Noconclusion

(4) If A is an $n \times n$ none invertible matrix then

(a) The linear system $AX = b$ is inconsistent for some $b \in R^n$.

(b) The linear system $AX = b$ is consistent only if $b = 0$.

(c) The linear system $AX = b$ is consistent for every $b \in R^n$.

(d) The linear system $AX = b$ has infinite solutions for some $b \in R^n$.

(5) If the coefficient matrix of a linear system $AX = b$ is $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 0 & 4 & 1 \end{bmatrix}$ then

(a) the system has infinitely many solutions

(b) the system is inconsistent

(c) the system must have a unique solution

(d) The system is consistent only if $b = 0$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 0 & 4 & 1 \end{bmatrix} \xrightarrow{-R_2} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 0 & 4 & 1 \end{bmatrix} \xrightarrow{-4R_2+R_3} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 5 \end{bmatrix} \xrightarrow{\frac{1}{5}R_3} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

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(6) If $A = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ then $A^{-1} =$

$-3e_1+e_2$

$E \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad E^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(a) $\begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

??

(7) Let $A = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$ then the (2,3) entry of A^2 is

$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$

(a) 1

(b) 2

(c) 3

(d) 4

$(1\ 0\ 1\ 0\ 1) \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} = (0+0+0+0+1) = 1$

(8) If $(A|b) = \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ -1 & 1 & -1 & 0 \\ -1 & 0 & \alpha & \beta \end{array}\right)$ is the augmented matrix of the system $AX = b$ then the system $AX = b$ has no solution if

$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ -1 & 1 & -1 & 0 \\ -1 & 0 & \alpha & \beta \end{array}\right] \xrightarrow[R_1+R_3]{R_1+R_2} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2+\alpha & 1+\beta \end{array}\right]$

(a) $\alpha = -2, \beta = -1$ $\quad 2\alpha \neq \beta+1$

(b) $\alpha = -2, \beta \neq -1$ $\quad \alpha = -2,\ \beta \neq -1$

(c) $\alpha \neq -2, \beta \neq -1$

(d) $\alpha \neq -2, \beta = -1$

$2+\alpha = 0$

$\boxed{-2 = \alpha}$

$1+\beta \neq 0$ $\quad \boxed{\beta \neq -1}$

(9) If $(A|b) = \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ -1 & 1 & -1 & 0 \\ -1 & 0 & \alpha & \beta \end{array}\right)$ is the augmented matrix of the system $AX = b$ then the system $AX = b$ has exactly one solution if

$2+\alpha \neq 0$

(a) $\alpha = -2, \beta = -1$ $\quad 2+\alpha = 1+\beta$

$\boxed{-2 \neq \alpha}$

(b) $\alpha = -2, \beta \in (-\infty, \infty)$ $\quad \alpha \neq -2$

$\boxed{\beta \in R}$

(c) $\alpha \neq -2, \beta \in (-\infty, \infty)$

(d) $\alpha \neq -2, \beta \neq -1$

(10) If $(A|b) = \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ -1 & 1 & -1 & 0 \\ -1 & 0 & \alpha & \beta \end{array}\right)$ is the augmented matrix of the system $AX = b$ then the system $AX = b$ has infinitely many solution if

$2+\alpha = 1+\beta$

$\alpha = -1, \quad \beta = -1$

(a) $\alpha = -2, \beta = -1$

$2 + \alpha = 0$

$\boxed{\alpha = -2}$

(b) $\alpha = -2, \beta \neq -1$

$1 + \beta = 0$

(c) $\alpha \neq -2, \beta \neq -1$

$\boxed{\beta = -1}$

(d) $\alpha \neq -2, \beta = -1$

(11) If $(A|b) = \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ -1 & 1 & -1 & 0 \\ -1 & 0 & \alpha & \beta \end{array}\right)$ is the augmented matrix of the system $AX = b$ then the system $AX = b$ is consistent if

$\alpha + 2 = \beta + 1$

$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2+\alpha & 1+\beta \end{array}\right]$

(a) $\alpha = -2, \beta = -1 \text{or } \alpha \neq -2, \beta \in (-\infty, \infty)$

(b) $\alpha = -2, \beta \neq -1 \text{or } \alpha \in (-\infty, \infty), \beta = -1$

(c) $\alpha \neq -2, \beta \neq -1 \text{or } \alpha \in (-\infty, \infty), \beta = -1$

$2 + \alpha \neq 0$

(d) $\alpha \neq -2, \beta = -1$ or $\alpha \in (-\infty, \infty), \beta = -1$

$\boxed{\beta \in R}$

$\boxed{\alpha \neq -2}$

$\alpha \neq -2$

$\beta \in R$

$\beta = -1$

Q3:(15 points)(a) Let A, B be symmetric matrices and $AB = BA$. Show AB is symmetric

A is symmetric $\Rightarrow A = A^T$ non-singular

B is symmetric $\Rightarrow B = B^T$ non-singular

AB is symmetric

$\Rightarrow (AB)^T = B^T A^T$

$= BA$

So $AB = BA$ is true

So AB is symmetric

(b) Let A be a square $n \times n$ nonsingular matrix. Show that $adj(adj(A)) = |A|^{n-2}A$

Let A $n \times n$ nonsingular

$adj(adj(A)) \Rightarrow$

But $adj(A)$ form

$A^{-1} = \frac{1}{|A|} adj(A)$

$\Rightarrow adj(A) = A^{-1}|A|$

So

$adj(adj(A)) \Rightarrow adj(A^{-1}|A|)$

$= (A^{-1}|A|)^{-1} |A^{-1}|A||$

$, (A^{-1})^{-1} = A$

$= |A|^{-1}(A^{-1})^{-1} |A^{-1}| |A|^n = |A|^{-1} A |A|^{-1} |A|^n$

$= |A|^{n-2} A$ #

$adj(A) = |A| I) A^{-1}$

$adj(A) = |A| A^{-1}$

$adj(adj A) =$

$adj(|A| A^{-1})$

$= ||A| A^{-1}| (|A| A^{-1})^{-1}$

$|A|^n |A^{-1}| A |A|^{-1}$ $(A^{-1})^{-1} = A$

$= \frac{|A|^n A}{|A|^2}$

$= |A|^{n-2} A$

$adj A = |A| A^{-1}$

$adj(adj A) = adj(|A| A^{-1})$

$adj(|A| A^{-1})$

$||A| A^{-1}| (|A| A^{-1})^{-1}$

$\frac{|A|^n}{|A|} \cancel{|A|} = A ||A||^{-1}$

$|A|\cancel{|B|} = I$ $|B| \neq 0$

$\cancel{|A| = \frac{|I|}{|B|}} \Rightarrow \cancel{|A| = \frac{1}{|B|}}$ $A^{-1}AB = A^{-1}I$

$B = A^{-1}$

(c) Let A, B be two square $n \times n$ matrices such that $AB = I$. Show that $B = A^{-1}$. ~~easy~~

let $\Rightarrow A$ $n \times n$ nonsingular { any A^{-1}

let $\Rightarrow B$ $n \times n$ nonsingular

$(AB = I)$ multiplying by A^{-1} from left

and $(A^{-1}A)B = (A^{-1}I) \Rightarrow I.B = A^{-1} \Rightarrow \boxed{B = A^{-1}}$

and $(AB = I)$ by B^{-1} from right

$A(BB^{-1}) = IB^{-1}$

$AI = B^{-1} \Rightarrow \boxed{A = B^{-1}}$

~~$AB = I \Rightarrow |AB| = |I|$~~ ~~$|I| = 1$~~

~~$\Rightarrow |A||B| = 1 \Rightarrow |B| = \frac{1}{|A|}$~~ ~~$\neq |B| \neq |A|$~~

(d) Let the system $Ax = 0$ have a nonzero solution. If the system $Ax = b$ is consistent, prove that $Ax = b$ has infinitely many solutions

$AX = 0$

let A non Singular

homogenious system

always consistant

So A is always consistant

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$AX = b \Rightarrow$ So let $AX = 0$

has unique Solution

Because it is nonsingular

$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$

$AX = 0$ has non zero Solution

let the Solution is $\begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix}$

$A \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix} = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$

$\Rightarrow \alpha_1 a_1 + \alpha_2 a_2 + \cdots + \alpha_n a_n = b$

So we can written b As linear combination of A

So A has infinty many solution

Q4(a) :(10 points) Find LU decomposition of the matrix $A = \begin{bmatrix} 2 & 4 & 2 \\ 1 & 5 & 2 \\ 0 & -3 & 9 \end{bmatrix}$

to find U ⇒

$\begin{bmatrix} 2 & 4 & 2 \\ 1 & 5 & 2 \\ 0 & -3 & 9 \end{bmatrix} \xrightarrow{-\frac{1}{2}R_1+R_2} \begin{bmatrix} 2 & 4 & 2 \\ 0 & 3 & 1 \\ 0 & -3 & 9 \end{bmatrix} \xrightarrow{R_2+R_3} \begin{bmatrix} 2 & 4 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 10 \end{bmatrix}$

upper is $= \begin{bmatrix} 2 & 4 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 10 \end{bmatrix}$

$, A = LU$

and L is $E_2E_1\begin{pmatrix} 2 & 4 & 2 \\ 1 & 5 & 2 \\ 0 & -3 & 9 \end{pmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$ lower

$A = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 4 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 10 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 2 \\ 1 & 5 & 2 \\ 0 & -3 & 9 \end{bmatrix}$ lower is ⇒

(b) Use LU decomposition of the matrix A in part (a) to solve the system $AX = b$, where $b = (1, 1, 1)^t$

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$A\,X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $LU\,X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

let $y = UX$

⇒ $\boxed{UX = y}$

and

$Ly = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ so ⇒ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ \frac{1}{2} & 1 & 0 & 1 \\ 0 & -1 & 1 & 1 \end{array}\right]$ y

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$\boxed{y_1 = 1}$, $\frac{1}{2}y_1 + y_2 = 1$ ⇒ $\frac{1}{2} + y_2 = 1$

$\boxed{y_2 = \frac{1}{2}}$

$-y_2 + y_3 = 1$ ⇒ $-\frac{1}{2} + y_3 = 1$ ⇒ $\boxed{y_3 = \frac{3}{2}}$

Solution of y is $(1, \frac{1}{2}, \frac{3}{2})$

and $UX = y$

$\left[\begin{array}{ccc|c} 2 & 4 & 2 & 1 \\ 0 & 3 & 1 & \frac{1}{2} \\ 0 & 0 & 10 & \frac{3}{2} \end{array}\right]$ ↑

$10x_3 = \frac{3}{2}$

$\boxed{x_3 = \frac{3}{20}}$

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$3x_2 + x_3 = \frac{1}{2}$

$3x_2 + \frac{3}{20} = \frac{1}{2}$ ⇒ $3x_2 = \frac{1}{2} - \frac{3}{20}$ ⇒ $3x_2 = \frac{10}{20} - \frac{3}{20}$

$3x_2 = \frac{7}{20}$ ⇒ $\boxed{x_2 = \frac{7}{60}}$

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Q4.b) $2x_1 + 4x_2 + 2x_3 = 1$

$2x_1 + \overset{2}{\cancel{4}} * \frac{7}{\underset{30}{\cancel{60}}} + 2 * \frac{3}{20} = 1$

$2x_1 + \frac{2 \times 14}{2 \times 30} + \frac{3 \times 6}{3 \times 20} = 1$

$2x_1 + \frac{28}{60} + \frac{18}{60} = 1$

$2x_1 = 1 - \frac{46}{60}$

$\begin{array}{r} 1 \\ 28 \\ 18 \\ \hline 46 \end{array}$

$\cancel{2x_2 =}$

$2x_1 = \frac{60}{60} - \frac{46}{60} = \frac{14}{60}$

$\begin{array}{r} 5\ 10 \\ \cancel{60} \\ 46 \\ \hline 14 \end{array}$

$\boxed{x_1 = \frac{7}{60}}$

Solution of x is $\left(\frac{7}{60}, \frac{7}{60}, \frac{3}{20}\right)$