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Birzeit University
Mathematics Department
Math 234

First Exam | Summer Semester 2019

Student Name: ....هند مؤيد بشتكي.... Number:..1181401...Section: ...3....

Question 1 (2 points each). Circle the most correct answer

(1) (..F..) If A is a 3×4-matrix, $b \in \mathbb{R}^3$, and the system $Ax = b$ is consistent, then $Ax = b$ has a unique solution.

(2) (..F..) If A is a singular matrix, then $\text{adj}(A)A = 0$.

$A\ \text{adj}(A) = \det(A)\, I$

$A\ \text{adj}(A) = 0$

(3) (..F..) If the matrix B is obtained from A by multiplying a row of A by 3, then $\det(A) = 3\det(B)$.

$E_{11} A = B$

(4) (..T..) If A is a 3×3-matrix and $A \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, then A is singular.

(5) (..F..) If A is a singular 3×3-matrix, then the reduced row echelon form of A has 2 rows of zeros.

(6) (..T..) If x_0 is a solution of the homogeneous system $Ax = 0$, and x_1 is a solution of the nonhomogeneous system $Ax = b$. Then $x_1 + x_0$ is a solution of the system $Ax = b$.

(7) (..T..) Any two $n \times n$-nonsingular matrices are row equivalent.

(8) (..F..) If A, B are 3×3-matrices, $|A| = -2$ and $|B| = 4$, then $|-3A^{-1}B^T| = 54$

$-3\,|A^{-1}|\,|B^T| = \frac{-3}{|A|}\,|B|$

$6 = \frac{(-3)(4)}{(-2)}$

(9) (..F..) Let A be a 3×4 matrix which has a column of zeros, and let B be a 4×4 matrix, then AB has a column of zeros.

$[\ \ ]_{3\times4}\ [\ \ ]_{4\times4} = [\ \ ]_{3\times4}$

(10) (..T..) If A is a singular matrix and U is the row echelon form of A, then $\det(U) = 0$.

(11) (..T..) If $A = LU$ is the LU-factorization of a matrix A, and A is nonsingular, then L and U are both nonsingular.

(12) (..T..) If A is a 3×3-matrix and the system $Ax = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$ has a unique solution, then the system

non-singular.

$Ax = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ has only the zero solution.

(13) (..T..) If A is singular and B is nonsingular $n \times n$-matrices, then AB is singular.

(14) (..T..) If A is a singular matrix, then A^T is singular.

(15) (..F..) If A is row equivalent to B, then $\det(A) = \det(B)$.

(16) (..T..) The vector $(0, 0, 0)^T$ is a linear combination of the vectors $(1, 2, 3)^T, (1, 4, 1)^T, (2, 3, 1)^T$

(17) (F..F..) If A, B are $n \times n$ symmetric matrices then AB is symmetric.

$A^T = A$ $\quad (AB)^T = B^T A^T = BA$

(18) (...T..) If A is a 3×5 matrix, then the system $Ax = 0$ has a nonzero solution.

(19) (..F...) If y, z are solutions to the system $Ax = 0$, then any linear combination of y, z is also a solution to $Ax = 0$.

infinetly nonzero

$A y = 0$
$A z = 0$

(20) (..T...) If A is a symmetric $n \times n$-matrix and P any $n \times n$-matrix, then $P^T AP$ is a symmetric matrix.

$A^T = A$ $\quad (P^T A P)^T = P^T A P$

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(21) (..F...) If A is an $n \times n$-matrix with positive entries, then $\det(A) \geq 0$.

$\begin{bmatrix} 1 & 4 \\ 8 & 2 \end{bmatrix} = 2 - (4)(8) = -[\ ]$

(22) (...T..) If A is a symmetric $n \times n$-matrix and P any $n \times n$-matrix, then $P^T AP$ is a symmetric matrix.

$A^T = A$

(23) (...T..) If A is symmetric and skew symmetric then $A = 0$. (A is skew symmetric if $A = -A^T$).

$A^T = A \;\&\; A^T = -A$

(24) (....T..) Let A be a square nonsingular $n \times n$ matrix. If $|adjA| = |A|$ then A is a 2×2-matrix.

$\boxed{n=2} \leftarrow |adj A| = |A|^{n-1}$ $\quad A \cdot adj A = \det A \, I_{2\times 2}$

(25) (...T..) An $n \times n$-matrix A is nonsingular if and only if A is a product of elementary matrices.

Question 2 (2 points each). Circle the most correct answer

(1) Let A be a 4×3-matrix with $a_2 = a_3$. If $b = a_1 + a_2 + a_3$, where a_j is the jth column of A, then the system $Ax = b$ has

(a) no solution.

(b) exactly one solution.

(c) infinitely many solutions.

(d) only 4 solutions.

$(a_1, a_2, a_3) \; X = (a_1 + a_2 + a_3)$

$X a_1 + 2X a_2 = a_1 + a_2 + a_3$

$X a_1 + 2X a_2 = a_1 + 2a_2$

(2) If $(A|b) = \left(\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 2 & -1 & 2 & 6 \\ 0 & 3 & 2 & 1 \end{array}\right)$ is the Augmented matrix of the system $Ax = b$ then the system has

$-2R_1 + R_2 \quad \left(\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & -3 & -2 & -2 \\ 0 & 3 & 2 & 1 \end{array}\right)$

$R_2 + R_3 \quad \left(\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & -3 & -2 & -2 \\ 0 & 0 & 0 & -1 \end{array}\right)$

No Sol.

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(a) no solution

(b) exactly one solution

(c) exactly 2 solutions

(d) infinitely many solutions

(3) If A is a nonsingular and symmetric matrix, then

(a) ~~A^{-1} is singular and symmetric~~

(b) A^{-1} is nonsingular and symmetric

(c) A^{-1} is nonsingular and not symmetric

(d) A^{-1} is ~~singular and not~~ symmetric

non Sing.

$\boxed{A^T = A}$

$A A^{-1} = I$

$A^T (A^T)^{-1} = I$

$(A^{-1})^T = (A^T)^{-1} = A^{-1}$

(4) If $A = \begin{pmatrix} 1 & -2 & 5 \\ 4 & -5 & 8 \\ -3 & 3 & a \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$, then the system $Ax = b$ has infinitely many solutions if and only if

(a) $b_3 = b_1 - b_2$ and $a = -3$

(b) ~~$a \neq -3$~~

(c) $b_3 = b_1 - b_2$ or $a = -3$

(d) $a = -3$

$-4R_1+R_2$, $3R_1+R_3$ $\begin{pmatrix} 1 & -2 & 5 \\ 0 & 3 & -12 \\ 0 & -3 & 15+a \end{pmatrix}$

$R_2+R_3 \to \begin{pmatrix} 1 & -2 & 5 \\ 0 & 3 & -12 \\ 0 & 0 & 3+a \end{pmatrix}$

$3+a = 0$ and $b_3 = 0$

(5) The adjoint of the matrix $\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$ is

$\det(A) = 3+2 = 5$

(a) $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & 2 \\ -1 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix}$

$A \cdot \operatorname{adj}(A) = \det(A)\, I_{2\times 2}$

$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ -1 & 3 & 0 & 1 \end{array}\right) \xrightarrow{R_1+R_2} \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 5 & 1 & 1 \end{array}\right)$

$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & 1/5 & 1/5 \end{array}\right) \xrightarrow{-2R_2+R_1} \left(\begin{array}{cc|cc} 1 & 0 & 3/5 & -2/5 \\ 0 & 1 & 1/5 & 1/5 \end{array}\right)$

$A^{-1} 5 b$

$\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

(6) An $n \times n$ matrix A is nonsingular if and only if

(a) $Ax = 0$ has nonzero solutions

(b) there exists a matrix B such that $AB = I$

(c) $|A| = 0$ X

(d) All of the above

(7) If A is a singular matrix, then the system $Ax = 0$

(a) has nonzero solutions

(b) has only the zero solution

(c) is inconsistent

(d) none of the above

(8) Let A be an $n \times n$-matrix in reduced row echelon form and $A \neq I$, then

Singular

(a) $\det(A) = 1$

(b) A is the zero matrix

(c) The system $Ax = 0$ has infinitely many solutions

(d) A is nonsingular

(9) Let A be an $n \times n$-matrix such that $A^T = A^{-1}$, then $\det(A) =$

(a) 1

(b) -1

(c) ± 1

(d) 0

$\det(A^T) = \det(A^{-1})$

$\det(A) = \dfrac{1}{\det(A)}$

$\det(A) = \pm 1$

(10) If E is an elementary matrix of type I, then A^T is

(a) an elementary matrix of type III

(b) an elementary matrix of type II

(c) an elementary matrix of type I

(d) not an elementary matrix

(11) One of the following matrices is in reduced row echelon form

rref

(a) $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

(12) If A is a nonsingular $n \times n$-matrix, then

(a) The system $Ax = 0$ has a nontrivial (nonzero) solution.

(b) $\det(A) = 1$

(c) There is an elementary matrix E such that $A = E$.

(d) There is a nonsingular matrix C such that $A = CI$.

$(\det B)^2 = \det B = 0$

$\det B\,(\det B - 1) = 0$

$\det B = 1, 0$

(13) If B is a 3×3 matrix such that $B^2 = B$. One of the following is always true

(a) $B^5 = B$.

(b) $B = I$.

(c) ~~B is nonsingular~~

(d) $\det(B) \neq 0$.

$B\,B = B$

$\det(B)\det(B) = \det(B)$

$\det(B) = 1$

$B^2 = B$

$B^3 = B^2 = B$

$B^4 = B^2 = B$

$B^5 = B.$

(14) If A and B are $n \times n$ matrices such that $Ax = Bx$ for some non zero $x \in \mathbb{R}^n$. Then

(a) $A - B$ is singular.

(b) A and B are nonsingular.

(c) A and B are singular.

(d) none of the above

$A^{-1}(AB = A\,C)$

$|A|$ → non Singular $\boxed{B = C}$

(15) If $AB = AC$, and $|A| \neq 0$, then

$\boxed{B=C}$ A ~~(a) $B = C$.~~ اكيدة

~~(b) $C \leq B$~~

~~(c) $B \neq C$~~

~~(d) $A = C$~~

$\cancel{\det(A)}\ \det(B) = \cancel{\det(A)} \cdot \det(C)$

$\det(B) = \det(C)$

$B=C$

Question 3 (5 points). Let A be an $n \times n$-matrix. Prove that if $AA^T = A$, then A is symmetric and $A^2 = A$.

$A\ A^T = A$

6 $A^T = A$

$A^2 = A$

$\cancel{A^2 = A}$

$A = A\ A^T \longrightarrow (A)^T = (A\ A^T)^T$

$A^T = A\ A^T$

$\boxed{A^T = A}$

$\therefore$ A is Symmetric.

$A = A\ A^T = A\,(A)$

$\cancel{(A)^2 = (A\,A^T)}$

$\boxed{A = A^2}$

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Question 4 (20 points). (i) If $2A^{-1} = \begin{pmatrix} 2 & 1 \\ 2 & 4 \end{pmatrix}$, then $A =$

$A^{-1} = \begin{pmatrix} 4 & 2 \\ 4 & 8 \end{pmatrix}$

$\left(\begin{array}{cc|cc} 4 & 2 & 1 & 0 \\ 4 & 8 & 0 & 1 \end{array}\right) \xrightarrow{-R_1+R_2} \left(\begin{array}{cc|cc} 4 & 2 & 1 & 0 \\ 0 & 6 & -1 & 1 \end{array}\right)$

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$\xrightarrow{-\frac{1}{3}R_2+R_1} \left(\begin{array}{cc|cc} 4 & 0 & 4/3 & -1/3 \\ 0 & 6 & -1 & 1 \end{array}\right)$ $\frac{1}{3}$ $-\frac{1}{12}$ $-\frac{1}{6}$ $\frac{1}{6}$

$A = \left(\begin{array}{cc|cc} 1 & 0 & 1/3 & -1/12 \\ 0 & 1 & -\frac{1}{6} & 1/6 \end{array}\right)$

$A = \begin{pmatrix} 1/3 & -1/12 \\ -1/6 & 1/6 \end{pmatrix}$.

(ii) The LU-factorization of $A = \begin{pmatrix} 2 & 2 & -4 \\ 1 & 1 & 2 \\ -1 & -1 & 3 \end{pmatrix}$ is

$L = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

$U = \begin{pmatrix} 1 & 1 & -2 \\ 0 & 0 & 4 \\ 0 & 0 & 1 \end{pmatrix}$

2

$\frac{1}{2}R_1$

$\begin{pmatrix} 1 & 1 & -2 \\ 1 & 1 & 2 \\ -1 & -1 & 3 \end{pmatrix} \xrightarrow[R_1+R_3]{-R_1+R_2} \begin{pmatrix} 1 & 1 & -2 \\ 0 & 0 & 4 \\ 0 & 0 & 1 \end{pmatrix}$

$LU = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & -2 \\ 0 & 0 & 4 \\ 0 & 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 2 & 2 & -4 \\ 1 & 1 & 2 \\ -1 & -1 & 3 \end{pmatrix}$ ✓

(iii) Let $A = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 4 & 5 \\ 0 & 3 & 2 \end{pmatrix}$, then

~~$3(-7) + 2(-1) + (3)(2) = -21 - 2 + 6 = -17$~~

$A_{11} = -7$ | $A_{12} = -2$ | $A_{23} = -9$

$A_{22} = 6$ | $A_{21} = -2 - 6 = +8$ | $A_{32} = -13$

$A_{33} = 13$ | $A_{13} = 3$ | $A_{31} = -5 - 8 = -13$.

(a) $\operatorname{adj}(A) =$

~~$\begin{pmatrix} -7 & -2 & 3 \\ 8 & 6 & -13 \end{pmatrix}$~~

$\begin{pmatrix} -7 & 8 & -13 \\ -2 & 6 & -9 \\ 3 & -13 & 13 \end{pmatrix}$

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(b) $\det(A) =$

$a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = 3(-7) + 2(+1) + (2)(3)$

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$= -21 + 8 = \boxed{-13}$.

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(c) $A^{-1} =$

3

$\left[\begin{array}{ccc|ccc} 3 & -1 & 2 & 1 & 0 & 0 \\ 1 & 4 & 5 & 0 & 1 & 0 \\ 0 & 3 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-\frac{1}{3}R_1+R_2} \left[\begin{array}{ccc|ccc} 3 & -1 & 2 & 1 & 0 & 0 \\ 0 & 13/3 & 13/3 & -\frac{1}{3} & 1 & 0 \\ 0 & 3 & 2 & 0 & 0 & 1 \end{array}\right]$

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$\rightarrow \left[\begin{array}{ccc|ccc} 3 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -\frac{1}{13} & \frac{3}{13} & 0 \\ 0 & 3 & 2 & 0 & 0 & 1 \end{array}\right]$

$\xrightarrow[-3R_2 R_3]{R_2+R_1} \left(\begin{array}{ccc|ccc} 3 & 0 & 3 & 12/13 & 3/13 & 0 \\ 0 & 1 & 1 & -\frac{1}{13} & \frac{3}{13} & 0 \\ 0 & 0 & +1 & 3/13 & +9/13 & -1 \end{array}\right)$ $4/3$ $1/13$

$\begin{bmatrix} 7/13 & & \\ 2/13 & -6/13 & 1 \\ -3/13 & 9/13 & -1 \end{bmatrix}$