MATH 234
FIRST HOUR EXAM

Student Name: Key I Student Number:

Instructor: Section:

Question 1. (20 points) Answer by true or false:

1. F If a system of linear equations is undetermined, then it must have infinitely many solutions.
2. T If A and B are $n \times n$ nonsingular matrices, then AB is also nonsingular.
3. F If A and B are 2×2 matrices that satisfy $AB = O$, then $A = O$ or $B = O$.
4. F The product of two elementary matrices of the same size is an elementary matrix.
5. F The sum of two $n \times n$ nonsingular matrices is also nonsingular.
6. T If a matrix A is symmetric, then the matrix A^T is also symmetric.
7. T If a matrix A is row equivalent to I, then A is nonsingular.
8. T If a matrix A is nonsingular, then the matrix A^T is also nonsingular.
9. T The inverse of an elementary matrix is also an elementary matrix.
10. F The method of solving a linear system by reducing its augmented matrix to reduced row echelon form is called Gaussian elimination.
11. F If $A\mathbf{x} = \mathbf{b}$ is an overdetermined and consistent linear system, then it must have infinitely many solutions.
12. T Any two nonsingular matrices are row equivalent.
13. T A homogeneous system can have a nontrivial solution.
14. F If A and B are $n \times n$ matrices such that $AB = BA$, then A and B are both nonsingular.
15. F If $|A| = 0$, then A must have two identical rows or two identical columns.
16. T If A is an $n \times n$ matrix, and $A\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ has a unique solution, then $A\mathbf{x} = \mathbf{b}$ has a unique solution for every $\mathbf{b} \in R^3$
17. F If a matrix is in row echelon form, then it is also in reduced row echelon form.
18. F It is possible for a singular matrix to be row equivalent to the identity matrix.
19. F If A is a 3×3 matrix, then $|-2A| = -2|A|$.
20. T If A is an $n \times n$ matrix, then $|A^n| = |A|^n$

Question 2 (3 points each) Circle the most correct answer:

1. If A is a 3×5 matrix, then

   (a) $A\mathrm{x} = \mathrm{b}$ is consistent for every vector $b \in R^5$.
   (b) $A\mathrm{x} = \mathrm{b}$ is inconsistent for every vector $b \in R^5$.
   (c) $A\mathrm{x} = 0$ has infinitely many solutions.
   (d) $A\mathrm{x} = 0$ has only the trivial solution.

2. Let $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 0 \\ 1 & 8 & 1 \end{bmatrix}$. If we want to find the LU factorization of A, then $L =$

   (a) $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$
   (b) $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 8 & 1 \end{bmatrix}$
   (c) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & -2 & 1 \end{bmatrix}$
   (d) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & -8 & 1 \end{bmatrix}$

3. If $B = EA$, where E is an elementary matrix of type I, then

   (a) $|B| = |A|$
   (b) $|B| = -|A|$
   (c) $|B| = |E|$
   (d) $|B| = -|E|$

4. If A and B are $n \times n$ matrices such that A singular and B is nonsingular, then

   (a) $AB = O$
   (b) AB is singular
   (c) $A + B$ is singular
   (d) AB is nonsingular

5. Let $A = \begin{bmatrix} a & 1 & 1 \\ a & 2 & 0 \\ -2 & 2 & a \end{bmatrix}$. Then A is nonsingular if and only if

   (a) $a = 2$
   (b) a is any real number
   (c) $a = -2$
   (d) $a = \pm 2$

6. Let A be a 3×3 matrix and suppose that $A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. Then

(a) $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions
(b) $A\mathbf{x} = (1, 0, 0)^T$ has infinitely many solutions
(c) A is nonsingular
(d) None of the above

7. Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $B = \begin{bmatrix} a & 4d & a+g \\ b & 4e & b+h \\ c & 4f & c+i \end{bmatrix}$. If $|A| = 3$ then $|B| =$

(a) 12
(b) 6
(c) 3
(d) 24

8. If A is a 4×4 matrix with $\det(A) = 2$ then $\det(4A^{-1})$ is

(a) 2
(b) 8
(c) 264
(d) 128

9. Let $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$. Then the adjoint of A is

(a) $\begin{bmatrix} 1 & -3 \\ -4 & 2 \end{bmatrix}$
(b) $\begin{bmatrix} -1 & 4 \\ 3 & -2 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & -4 \\ -3 & 2 \end{bmatrix}$
(d) $\begin{bmatrix} -1 & 3 \\ 4 & -2 \end{bmatrix}$

10. If $\det(A) \neq 0$ then

(a) A is nonsingular
(b) $A\mathbf{x} = \mathbf{0}$ has only the trivial solution
(c) A is row equivalent to I
(d) All of the above

Question 3.(10 points) Suppose that $[A|\mathbf{b}] = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 2 & 1 & -1 & 5 \\ 1 & -1 & a & b \end{array}\right]$ is the augmented matrix of a linear system.

For what values of a and b does the system have

1. a unique solution.
2. no solution.
3. infinitely many solutions.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 3 & -3 & 1 \\ 0 & 0 & a-1 & b-2 \end{array}\right]$$

1. $a \neq 1$

2. $a = 1$ & $b \neq 2$

3. $a = 1$ & $b = 2$

Question 4.(12 points) Use Guauss-Jordan reduction to solve the system

$$\begin{aligned} x_1 &+ x_2 + 2x_3 + x_4 = 5 \\ 2x_1 &- x_2 + x_3 + 8x_4 = 4 \\ 4x_1 &+ x_2 + 5x_3 - 2x_4 = 5 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 5 \\ 2 & -1 & 1 & 8 & 4 \\ 4 & 1 & 5 & -2 & 5 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 5 \\ 0 & -3 & -3 & 6 & -6 \\ 0 & -3 & -3 & -6 & -15 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 5 \\ 0 & 1 & 1 & -2 & 2 \\ 0 & 0 & 0 & -12 & -9 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & 1 & 2 & 0 & \frac{17}{4} \\ 0 & 1 & 1 & 0 & \frac{7}{2} \\ 0 & 0 & 0 & 1 & \frac{3}{4} \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & \frac{3}{4} \\ 0 & 1 & 1 & 0 & \frac{7}{2} \\ 0 & 0 & 0 & 1 & \frac{3}{4} \end{array}\right]$$

let $x_3 = t \Rightarrow x_4 = \frac{3}{4}$, $x_2 = \frac{7}{2} - t$, $x_1 = \frac{3}{4} - t$

Sol. : $\left(\frac{3}{4} - t, \frac{7}{2} - t, t, \frac{3}{4}\right)^T$, t any real #.

Question 5.(10 points) Let $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & -2 & -3 \end{bmatrix}$. Find

1. The adjoint of A.
2. The determinant of A.
3. The inverse of A.

1. $\begin{bmatrix} -1 & -4 & 3 \\ -2 & -2 & 2 \\ -1 & -2 & 1 \end{bmatrix}^T$

2. $|A| = 2$

3. $A^{-1} = \frac{1}{2} \operatorname{adj} A$

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   (b) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & -8 & 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

   (d) $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 8 & 1 \end{bmatrix}$

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   (b) $A + B$ is singular
   (c) $AB = O$
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