MATH234

Tareq Shannk 1181404

Tareq Shannak
1181404

Q1)

$$\left(\begin{array}{cccc|c} 1 & -1 & 3 & 2 & 1 \\ -1 & 1 & -2 & 1 & -2 \\ 2 & -2 & 7 & 7 & 1 \end{array}\right)$$

$$\xrightarrow[-2R_1+R_3]{R_1+R_2} \left(\begin{array}{cccc|c} 1 & -1 & 3 & 2 & 1 \\ 0 & 0 & 1 & 3 & -1 \\ 0 & 0 & 1 & 3 & -1 \end{array}\right)$$

$$\xrightarrow{-R_2+R_3} \left(\begin{array}{cccc|c} 1 & -1 & 3 & 2 & 1 \\ 0 & 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

let $X_4 = \alpha$, $X_3 = -1-3\alpha = -(1+3\alpha)$

let $X_2 = \beta$, $X_1 = 1-2\alpha+3(1+3\alpha)+\beta$

$= 4+\beta+7\alpha$

Solution Set $= \left\{ (X_1, X_2, X_3, X_4) = \left((4+\beta+7\alpha), \beta, (-(1+3\alpha)), \alpha\right) : \alpha, \beta \in R \right\}$

Q2)

$$\left(\begin{array}{ccc|c} 3 & -1 & \alpha & 1 \\ 1 & 3 & 2 & -b \\ 1 & -2 & 2 & 4 \end{array}\right)$$

$$\xrightarrow[-R_3+R_2]{-2R_3+R_1} \left(\begin{array}{ccc|c} 1 & 3 & \alpha-4 & -7 \\ 0 & 5 & 0 & -b-4 \\ 1 & -2 & 2 & 4 \end{array}\right)$$

$$\xrightarrow{R_1 \Leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & -2 & 2 & 4 \\ 0 & 5 & 0 & -b-4 \\ 1 & 3 & \alpha-4 & -7 \end{array}\right)$$

$$\xrightarrow[\frac{1}{5}R_2]{-R_1+R_3} \left(\begin{array}{ccc|c} 1 & -2 & 2 & 4 \\ 0 & 1 & 0 & \frac{-(b+4)}{5} \\ 0 & 5 & \alpha-6 & -11 \end{array}\right)$$

$$\xrightarrow{-5R_2+R_3} \left(\begin{array}{ccc|c} 1 & -2 & 2 & 4 \\ 0 & 1 & 0 & -\frac{(b+4)}{5} \\ 0 & 0 & \alpha-6 & b-7 \end{array}\right)$$

$\boxed{1}$ $\alpha = 6$, $b \neq 7$

$\boxed{2}$ $\alpha \neq 6$

$\boxed{3}$ $\alpha = 6$, $b = 7$

Q3)

$$A = \begin{pmatrix} 1 & 4 & 5 \\ -3 & -1 & -2 \\ 2 & 3 & 4 \end{pmatrix}$$

$$(A|I) = \left(\begin{array}{ccc|ccc} 1 & 4 & 5 & 1 & 0 & 0 \\ -3 & -1 & -2 & 0 & 1 & 0 \\ 2 & 3 & 4 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[-2R_1+R_3]{3R_1+R_2} \left(\begin{array}{ccc|ccc} 1 & 4 & 5 & 1 & 0 & 0 \\ 0 & 11 & 13 & 3 & 1 & 0 \\ 0 & -5 & -6 & -2 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{2R_3+R_2} \left(\begin{array}{ccc|ccc} 1 & 4 & 5 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 2 \\ 0 & -5 & -6 & -2 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[-4R_2+R_1]{5R_2+R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 5 & -4 & -8 \\ 0 & 1 & 1 & -1 & 1 & 2 \\ 0 & 0 & -1 & -7 & 5 & 11 \end{array}\right)$$

$$\xrightarrow[R_3+R_2]{R_3+R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 1 & 3 \\ 0 & 1 & 0 & -8 & 6 & 13 \\ 0 & 0 & -1 & -7 & 5 & 11 \end{array}\right)$$

$$\xrightarrow{(-1)R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 1 & 3 \\ 0 & 1 & 0 & -8 & 6 & 13 \\ 0 & 0 & 1 & 7 & -5 & -11 \end{array}\right)$$

$$= (I \mid A^{-1})$$

A is nonsingular and its inverse exists.

$$A^{-1} = \begin{pmatrix} -2 & 1 & 3 \\ -8 & 6 & 13 \\ 7 & -5 & -11 \end{pmatrix}$$

Q4)

$A_{n \times n}$

1) $\det(A) \neq 0$

2) A is row equivalent to I_n

3) The system $Ax = 0$ has only the zero solution ($x = 0$).

4) A is product of nonsingular matrices.

Q5)

S is nonsingular $\Rightarrow \det(S) = K \neq 0$

$\det(S^{-1}) = \frac{1}{K}$

$A = S^{-1} B S \Rightarrow \det(A) = \det(S^{-1} B S)$

$\det(A) = \det(S^{-1}) \det(B) \det(S)$

$\det(A) = \cancel{\left(\frac{1}{K}\right)} (\det(B)) \cancel{(K)} = \det(B)$ //

Q 6)

$$A = \begin{pmatrix} 2-K & -1 \\ -1 & 2-K \end{pmatrix}$$

when A is singular, $\det(A) = 0$

$$|A| = (2-K)^2 - 1 = 4 - 4K + K^2 - 1 = K^2 - 4K + 3$$

$$(K-3)(K-1) = 0 \Rightarrow \boxed{K = 1, 3}$$

Q 7)

$$\det\begin{pmatrix} 2g & 2h & 2i \\ -d & -e & -f \\ 2a & 2b & 2c \end{pmatrix} = -4 \det\begin{pmatrix} g & h & i \\ d & e & f \\ a & b & c \end{pmatrix}$$

$$= 4 \det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 4 \times 2 = 8$$

Q8) $A = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 4 & 5 \\ 0 & 3 & 2 \end{pmatrix}$

$\boxed{1}$ $\text{adj}(A) = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}^T$

$A_{11} = (-1)^2 \begin{vmatrix} 4 & 5 \\ 3 & 2 \end{vmatrix} = 8 - 15 = -7$

$A_{21} = -\begin{vmatrix} -1 & 2 \\ 3 & 2 \end{vmatrix} = -(-2-6) = 8$

$A_{31} = \begin{vmatrix} -1 & 2 \\ 4 & 5 \end{vmatrix} = -5 - 8 = -13$

$A_{12} = -\begin{vmatrix} 1 & 5 \\ 0 & 2 \end{vmatrix} = -2$

$A_{22} = \begin{vmatrix} 3 & 2 \\ 0 & 2 \end{vmatrix} = 6$

$A_{32} = -\begin{vmatrix} 3 & 2 \\ 1 & 5 \end{vmatrix} = -(15-2) = -13$

$A_{13} = \begin{vmatrix} 1 & 4 \\ 0 & 3 \end{vmatrix} = 3$

$A_{23} = -\begin{vmatrix} 3 & -1 \\ 0 & 3 \end{vmatrix} = -9$

$A_{33} = \begin{vmatrix} 3 & -1 \\ 1 & 4 \end{vmatrix} = 13$

$\text{adj}(A) = \begin{pmatrix} -7 & -2 & 3 \\ 8 & 6 & -9 \\ -13 & -13 & 13 \end{pmatrix}^T = \begin{pmatrix} -7 & 8 & -13 \\ -2 & 6 & -13 \\ 3 & -9 & 13 \end{pmatrix}$

$\boxed{2}$ $\det(A) = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$= (3)(-7) + (-1)(-2) + (2)(3)$

$= -21 + 2 + 6 = -13$

$\boxed{3}$ $A^{-1} = \dfrac{\text{adj}(A)}{\det(A)}$

$= \dfrac{-1}{13} \begin{pmatrix} -7 & 8 & -13 \\ -2 & 6 & -13 \\ 3 & -9 & 13 \end{pmatrix}$

$= \begin{pmatrix} \frac{7}{13} & \frac{-8}{13} & 1 \\ \frac{2}{13} & \frac{-6}{13} & 1 \\ \frac{-3}{13} & \frac{9}{13} & -1 \end{pmatrix}$