Birzeit University
Mathematics Department
Math 234

Homework 2 (Chapter 3) Second Semester 2019/2020
Deliver the solution as one pdf-file using ritaj messages. File name should be studentNumber-HW2-Math234.pdf
Deadline Thursday 14-5-2020 Tareq Shannak 1181404

Question 1. Mark each of the following statements by True or false

(1) (..✓..) If $v_1, v_2, \cdots, v_k$ are vectors in a vector space V, and $\text{Span}(v_1, v_2, \cdots, v_k) = \text{Span}(v_1, v_2, \cdots, v_{k-1})$, then $v_1, v_2, \cdots, v_k$ are linearly dependent.

(2) (..✓..) If A is an $m \times n$-matrix, then $\text{rank}(A) = \text{rank}(A^T)$.

(3) (..✓..) If A, B are row equivalnt matrices, then $R(A) = R(B)$ (A, B have the same raw space).

(4) (..✗..) The set $S = \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2 : x_1 x_2 = 0 \right\}$ is a subspace of $\mathbb{R}^2$

(5) (..✓..) If A is a 6×4-matrix, $\text{rank}(A) = 4$, b is in the column space of A, then the system $Ax = b$ has exactly one solution.

(6) (..✓..) If A is 3×5-matrix, $\text{rank}(A) = 3$, then the system $Ax = b$ has infinitely many solutions for every $b \in \mathbb{R}^3$.

(7) (..✓..) If A is a 5×4-matrix, and $Ax = 0$ has only the zero solution, then $\text{rank}(A) = 4$.

(8) (..✓..) $S = \{A \in \mathbb{R}^{3\times3} : A \text{ is upper triangular}\}$ is a subspace of $\mathbb{R}^{3\times3}$

(9) (..✓..) If S is a subspace of a vector space V, then $0 \in S$

✓ (10) Let A be a 2×4 matrix, and $\text{rank}(A) = 2$, then, the columns of A form a spanning set for $\mathbb{R}^2$.

Question 2 (2.5 points each)**.** Circle the most correct answer

(1) If A is a 4×4-matrix, and $Ax = 0$ has only the zero solution, then $\text{rank}(A) =$

- (a) 1
- (b) 2
- (c) 3
- (d) 4

(2) If $f_1, f_2, \cdots, f_n \in C^{n-1}[a, b]$ and $W[f_1, f_2, \cdots, f_n](x) = 0$ for all $x \in [a, b]$, then $f_1, f_2, \cdots, f_n$ are

- (a) linearly independent.
- (b) linearly dependent
- (c) form a spanning set for $C^{n-1}[a, b]$
- (d) none of the above

(3) If the columns of $A_{n \times n}$ are linearly independent and $b \in \mathbb{R}^n$, then the system $Ax = b$ has

- (a) no solution
- (b) exactly one solution
- (c) infinitely many solutions
- (d) none of the above

(4) if $\{v_1, v_2, \cdots, v_k\}$ is a spanning set for $\mathbb{R}^{2\times 3}$, then

- (a) $k = 6$
- (b) $k \geq 6$
- (c) $k \leq 6$
- (d) $k > 6$

(5) If A is an $m \times n$ matrix, then

- (a) $\text{rank}(A) \leq m$
- (b) $\text{rank}(A) \leq n$
- (c) $\text{rank}(A) \leq \min\{m, n\}$
- (d) $\text{rank}(A) = m = n$

(6) If A is an $m \times n$-matrix, and columns of A are linearly independent, then

- (a) $m \leq n$
- (b) $n \leq m$
- (c) $m = n$
- (d) $m = n + 1$

(7) If A is a 3×5-matrix, rows of A are linearly independent, then

(a) $\text{rank}(A) = \text{nullity}(A)$

(b) $\text{rank}(A) = \text{nullity}(A) + 1$

(c) $\text{rank}(A) = \text{nullity}(A) + 2$

(d) $\text{rank}(A) = \text{nullity}(A) + 3$

(8) Let A be a 4×3 matrix, and $\text{rank}(A) = 3$

(a) The columns of A are linearly independent

(b) $\text{nullity}(A) = 0$

(c) The rows of A are linearly dependent

(d) All of the above

(9) If $\{v_1, v_2, v_3, v_4\}$ forms a spanning set for a vector space V, $\dim(V) = 3$, v_4 can be written as a linear combination of v_1, v_2, v_3, then

(a) v_1 can be written as a linear combination of v_2, v_3, v_4

(b) $\{v_1, v_2, v_3\}$ do not form a spanning set for V

(c) $\{v_1, v_2, v_3\}$ are linearly dependent

(d) $\{v_1, v_2, v_3\}$ is a basis for V

(10) The rank of $A = \begin{pmatrix} 1 & 4 & 1 & 2 & 1 \\ 2 & 6 & -1 & 2 & -1 \\ 3 & 10 & 0 & 4 & 0 \end{pmatrix}$ is

(a) 2

(b) 3

(c) 0

(d) 1

(11) If A is a 3×4 matrix, then

(a) The columns of A are linearly independent

(b) $Rank(A) = 3$

(c) $\text{nullity}(A) \geq 1$

(d) The rows of A are linearly dependent

(12) If A is an $n \times n$-matrix and for each $b \in \mathbb{R}^n$ the system $Ax = b$ has a unique solution, then

(a) A is nonsingular

(b) $\text{rank}(A) = n$

(c) $\text{nullity}(A) = 0$

(d) all of the above

(13) Let $E = [2 + x, 1 - x, x^2 + 1]$ be an ordered basis for P_3. If $p(x) = 3x^2 + 5x + 4$, then the coordinate vector of $p(x)$ with respect to E is

(a) $\begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix}$

(b) $\begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix}$

(c) $\begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix}$

(d) $\begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}$

(14) Let $S = \{ \begin{pmatrix} a+b \\ a+b \\ a+b \end{pmatrix} : a, b \in \mathbb{R}\}$. Then dimension of S equals

(a) 1
(b) 2
(c) 3
(d) 0

(15) If A is a 4×3 matrix such that $N(A) = \{0\}$, and b can be written as a linear combination of the columns of A, then

(a) The system $Ax = b$ has exactly one solution
(b) The system $Ax = b$ is inconsistent
(c) The system $Ax = b$ has infinitely many solutions
(d) None of the above

(16) Which of the following is not a basis for the corresponding space

(a) $\{5 - x, x\}$; P_2
(b) $\{(1, -1)^T, (2, -3)^T\}$; $\mathbb{R}^2$
(c) $\{x, 1 - x, 2x + 3\}$; P_3
(d) $\{(1, -1, -1)^T, (2, -3, 0)^T, (-1, 0, 2)^T\}$; $\mathbb{R}^3$

(17) If $\{v_1, v_2, v_3, v_4\}$ is a basis for a vector space V , then the set $\{v_1, v_2, v_3\}$ is

(a) linearly independent and not a spanning set for V.
(b) linearly dependent and not a spanning set for V.
(c) linearly independent and a spanning set for V.
(d) none of the above

Question 3 (6 points). If $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & 0 & 0 \\ -2 & -2 & 0 & 0 \end{pmatrix}$ and the row echelon form of A is

$$U = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

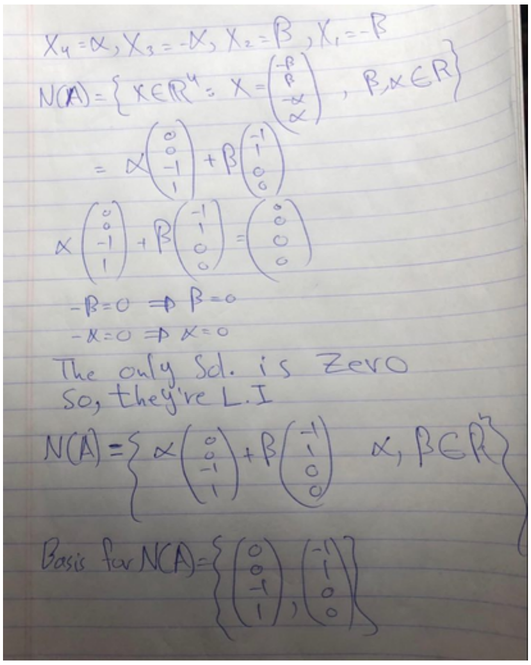
(a) Find a basis for the row space of A.

Basis for R(A) : {(1, 1, 1, 1), (0, 0, 1, 1)}

(b) Find a basis for the column space of A.

Basis for C(A) : {(1, -1, -2)T, (1, 0, 0)T}

(c) Find a basis for null space A



$X_4 = \alpha, X_3 = -\alpha, X_2 = \beta, X_1 = -\beta$

$N(A) = \left\{ X \in \mathbb{R}^4 : X = \begin{pmatrix} -\beta \\ \beta \\ -\alpha \\ \alpha \end{pmatrix}, \beta, \alpha \in R \right\}$

$= \alpha \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

$\alpha \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$-\beta = 0 \Rightarrow \beta = 0$

$-\alpha = 0 \Rightarrow \alpha = 0$

The only Sol. is Zero
So, they're L.I

$N(A) = \left\{ \alpha \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \alpha, \beta \in R \right\}$

Basis for $N(A) = \left\{ \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\}$

(d) Find Rank(A), Nullity(A).

Rank(A) = Dim(R(A)) = Dim(C(A)) = 2
Nullity(A) = 2