Birzeit University
Mathematics Department
Math 234

Homework 3 (Sections 4.1 and 6.1) Second Semester 2019/2020
Deliver the solution as one pdf-file using ritaj messages. File name should be studentNumber-HW3-Math234.pdf
Deadline Thursday 21-5-2020 Tareq Shannak 1181404

Question 1. Mark each of the following statements by True or false

(1) (..✓..) If $L : P_3 \to P_2$ is the linear transformation defined by $L(ax^2 + bx + c) = (a + b)x + (b + c)$, then $x^2 - x + 1$ is in $\ker(L)$.

(2) (..✓..) If $L : \mathbb{R}^2 \to \mathbb{R}^3$ is the linear transformation defined by $L(x, y)^T = (x - y, x + y, 2x - 2y)^T$, then $(2, 3, 4)$ is in $\text{Range}(L)$.

(3) (..✓..) If λ is an eigenvalue for a matrix A, then λ^2 is an eigenvalue for A^2.

(4) (..✗..) If $\lambda = 0$ is an eigenvalue for a matrix A, then $\det(A) \neq 0$.

(5) (..✓..) If λ is an eigenvalue of A with corresponding eigenvector x, then αx is an eigenvector of λ for every nonzero scalar α.

(6) (..✓..) If A is an $n \times n$-matrix, λ is an eigenvalue of A, then $\text{rank}(A - \lambda I) = n$.

(7) (..✗..) If A is a 2×2 matrix, $\text{trace}(A) = 0$ and 1 is an eigenvalue for A, then A is singular.

(8) (..✗..) Let $L : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $L(x, y) = (x - y, 1, 2x)$, then L is a linear transformation.

(9) (..✓..) Any Two similar matrices have the same eigenvalues.

Question 2. Circle the most correct answer

(1) Let $L : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation such that $L((1, 0, 0)^T) = (2, 2)^T$, $L((0, 2, 0)^T) = (-3, -5)^T$, $L((0, 0, 4)^T) = (-2, -3)^T$, then $L((1, -2, 4)^T) =$

(a) $(0, 0)^T$
(b) $(-3, -6)^T$
(c) $(3, 4)^T$
(d) $(-3, 0)^T$

(2) Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined as $L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_3 + x_1 \\ x_2 + x_1 \\ x_2 + x_1 \end{pmatrix}$, then $\dim(\ker(L)) =$

(a) 2
(b) 0
(c) 1
(d) 3

(3) Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined as $L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_3 + x_1 \\ x_2 + x_1 \\ x_2 + x_1 \end{pmatrix}$, then $\dim(\operatorname{Im}(L)) =$

(a) 3
(b) 0
(c) 1
(d) 2

(4) If a 4×4 matrix A has $1, -1, i$ as eigenvalues, then $\det(A) =$

(a) 0
(b) 1
(c) -1
(d) $-i$

(5) If A is a 3×3 singular matrix such that $\lambda_1 = 2, \lambda_2 = 3$ are eigenvalues of A, then $\operatorname{trace}(A) =$

(a) 18
(b) 36
(c) 6
(d) 0

trace(A) = 2+3+0 = 5 ?

(6) One of the following is not a linear transformation

(a) $T : R^2 \to R^3$ defined by $T(x, y) = (x, y, 0)$
(b) $T : R^2 \to R^3$ defined by $T(x, y) = (x, xy, x)$
(c) $T : R^2 \to R^3$ defined by $T(x, y) = (x, y, x - y)$
(d) $T : R^2 \to R^3$ defined by $T(x, y) = (0, y, x)$

(7) If A is a 3×3 matrix such that $Ax = 0$ has a nonzero solution, then one of the following might be the characteristic polynomial of A

(a) $x^2 - 2x$
(b) $x^3 - 2x$
(c) $x^3 - 2x + 1$
(d) $x^3 - 2$

(8) Let A be an upper triangular matrix, then the eigenvalues of A are

(a) the elements of the first column of A
(b) the elements of the main diagonal of A.
(c) the elements of the first row of A.
(d) none of the above

Question 3. Let $A = \begin{pmatrix} 2 & 0 & 0 \\ -1 & 4 & 0 \\ -3 & 6 & 2 \end{pmatrix}$.

(a) Find the eigenvalues of A.

$$P(\lambda) = |A - \lambda I| = \begin{vmatrix} 2-\lambda & 0 & 0 \\ -1 & 4-\lambda & 0 \\ -3 & 6 & 2-\lambda \end{vmatrix} = (2-\lambda)(4-\lambda)(2-\lambda) = 0$$

$$\lambda_1 = 4, \quad \lambda_2 = \lambda_3 = 2$$

(b) Find a basis for each eigenspace of A

$\lambda = 2$

$(A - 2I)X = 0$

$$\begin{bmatrix} 2-2 & 0 & 0 \\ -1 & 4-2 & 0 \\ -3 & -6 & 2-2 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ -1 & 2 & 0 \\ -3 & -6 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \boxed{X_3 = \beta} \quad X_2 = 0 \quad X_1 = 0$$

$$\Rightarrow E[\lambda = 2] = \left(\beta \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} : \beta \text{ scalar} \right)$$

$\lambda = 4$

$$\begin{bmatrix} 2-4 & 0 & 0 \\ -1 & 4-4 & 0 \\ -3 & -6 & 2-4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 0 \\ -1 & 0 & 0 \\ -3 & -6 & -2 \end{bmatrix} \begin{bmatrix} 3 & 6 & 2 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 6 & 2 \\ 0 & -18 & -6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 & 6 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \boxed{X_3 = \beta} \quad X_2 = \frac{-\beta}{3} \quad X_1 = 0$$

$$E[\lambda = 4] = \left(\beta \begin{pmatrix} 0 \\ -1/3 \\ 1 \end{pmatrix} \beta \text{ scalar} \right)$$

Basis for $N(A - \lambda I) =$

$$\left\{ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1/2 \\ 1 \end{pmatrix} \right\}$$