MATH 234
MIDTERM EXAM

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Section: S, M, W 9-10

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Question 1. (30%) Answer by true or false:

1. T If A is nonsingular then A^T is nonsingular.
2. F Span$\{1+x, 1-x\}$ is a subspace of P_2

   $c(1+x)+C_2(1-x)$

   $c+cx+C_2-C_2x$

   $=ax+b$
3. F Any singular matrix can be written as a product of elementary matrices.
4. T If A is row equivalent to B then both A and B are nonsingular.
5. T If span$\{\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3\} = R^3$ then span$\{\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3, \mathrm{x}\} = R^3$ for any $\mathrm{x} \in R^3$.
6. T If A is singular then adj(A) is singular.

   $x(C-C_2)$

   $C+C_2$

   $C_2+C_2=a$, $C_1+C_2=a$
7. T If A is $n \times n$ then $|A^n| = |A|^n$
8. T Suppose that $\{f_1, f_2, \ldots, f_n\} \subseteq C^{n-1}[a, b]$. If $W[f_1, \ldots, f_n] = 0$, where W denotes the Wronskien, then $f_1, f_2, \ldots, f_n$ are linearly dependent.
9. F Every diagonal matrix is nonsingular.
10. T $|AB| = |A||B|$ only when A or B is nonsingular.
11. F If E_1 and E_2 are elementary $n \times n$ matrices, then E_1E_2 is elementary.
12. T $(AB)C = A(BC)$ for all matrices A, B, and C when multiplication is allowed.
13. T If A and B are $n \times n$ matrices and A is singular, then AB is singuiar.
14. F If A is 3×3 then $|-2A| = -2|A|$.
15. T If A and B are symmetric $n \times n$ matrices then AB is symmetric.

   $A^T = A \quad B^T = B$

   $(AB)^T = B^T A^T = BA$

   $(BA)^T = A^T B^T = AB$
16. T If A is symmetric and nonsingular then A^{-1} is symmetric.
17. F If S is a set of vectors that are linearly independent in a vector space V then any nonempty subset of S is linearly independent
18. T If S is a set of vectors that are linearly independent in a vector space V then any subset of V containing S is linearly independent
19. T If S is a subspace of V then any set of vectors in S that spans S also spans V.
20. F If A is a singular $n \times n$ matrix, then $A\mathrm{x} = \mathrm{b}$ has infinitely many solutions for every vector $\mathrm{b} \in R^n$.

   sing ∞, inconsistent

Question 2 (42%) Circle the most correct answer:

1. Suppose that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly independent in V, then

   (a) The vectors $\{\mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_1 + \mathbf{v}_3, \mathbf{v}_2 + \mathbf{v}_3\}$ are linearly independent in V

   (b) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3\}$ are linearly independent in V

   (c) The vectors $\{\mathbf{v}_1, \mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_2 + \mathbf{v}_3\}$ are linearly independent in V

   (d) All of the above

2. One of the following is a subspace of $R^{n\times n}$

   (a) All singular $n \times n$ matrices

   (b) All upper triangular $n \times n$ matrices

   (c) All nonsingular $n \times n$ matrices

   (d) All triangular $n \times n$ matrices

3. Let $A = \begin{bmatrix} a & 1 & 1 \\ a & 2 & 0 \\ -2 & 2 & a \end{bmatrix}$. Then A is nonsingular if and only if

   $2a - a^2 + 2a + 4 = 0$

   $\boxed{-a^2 + 4a + 4 \neq 0}$

   (a) $a = 2$

   (b) a is any real number

   (c) $a = -2$

   (d) $a = \pm 2$

4. The set of vectors $\{(1, a)^T, (b, 1)^T\}$ is a spanning set for R^2 if

   $\begin{bmatrix} 1 & b \\ a & 1 \end{bmatrix}$ $1 - ab \neq 0$ $\boxed{1 \neq ab}$

   (a) $a \neq 1$ and $b \neq 1$

   (b) $ab \neq 1$

   (c) $ab = 1$

   (d) $a \neq b$

   3x3 3,1

   3 x 1

5. Let A be a 3×3 matrix and suppose that $A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. Then

   (a) $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions

   (b) $A\mathbf{x} = (1, 0, 0)^T$ has infinitely many solutions

   (c) A is nonsingular

   (d) None of the above

6. Suppose that a vector space V contains n linearly independent vectors, then

   (a) Any set containing more than n vectors is linearly dependent

   (b) If a set S spans V then S must contain at least n vectors

   (c) Any n vectors in V are linearly independent

   (d) If a set S spans V then S must contain at most n vectors

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad \det = 1$$

7. Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $B = \begin{bmatrix} a & 4d & a+g \\ b & 4e & b+h \\ c & 4f & c+i \end{bmatrix}$. If $|A| = 3$ then $|B| =$

(a) 12
(b) 6
(c) 3
(d) 24

$|A| = 6 \qquad |\operatorname{adj}(A)|$

8. If A is a 2×2 matrix with $\det(A) = 6$ then $\det(\operatorname{adj}(A))$ is

(a) 36
(b) 3
(c) 12
(d) 6

$A \operatorname{adj} A = I|A|$

$|A||\operatorname{adj} A| = |A|^2$

$\frac{36}{6} = 6$

$A \operatorname{adj} A = |A| I$

$\det(A)\det(\operatorname{adj} A) = \frac{36}{6} = 6$

9. If A is a 4×4 matrix with $\det(A) = 2$ then $\det(4A^{-1})$ is

(a) 2
(b) 8
(c) 264
(d) 128

$= 4^4 \left(\frac{1}{A}\right) \quad (4)^4 \, \frac{1}{2}$

10. One of the following is not a subspace of P_3

(a) $\{p \in P_3 \,|\, p(1) = p(-1)\}$
(b) $\{p \in P_3 \,|\, p(2) = 0\}$
(c) $\{p \in P_3 \,|\, p(2) = p(5)\}$
(d) $\{p \in P_3 \,|\, p(0) = 2\}$

11. One of the following sets is linearly independent in P_3

(a) $\{2x, 2-x, x^2\}$
(b) $\{1+x, 1-x, 1\}$
(c) $\{x, x^2, 2x+3x^2\}$
(d) $\{2, 2-x, x\}$

$c_1(2x) + c_2(2-x) + c_3(x^2) = ax^2 + bx + c$

$2c_1 - c_2 = b$

$2 = c$

$c_3 = a$

12. Let $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$. Then the adjoint of A is

(a) $\begin{bmatrix} 1 & -3 \\ -4 & 2 \end{bmatrix}$
(b) $\begin{bmatrix} -1 & 4 \\ 3 & -2 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & -4 \\ -3 & 2 \end{bmatrix}$
(d) $\begin{bmatrix} -1 & 3 \\ 4 & -2 \end{bmatrix}$

$\begin{bmatrix} 1 & -3 \\ -4 & 2 \end{bmatrix}^T$

$\begin{bmatrix} 1 & -4 \\ -3 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1 & -3 \\ -4 & 2 \end{bmatrix}$

13. If $\det(A) \neq 0$ then

(a) A is nonsingular
(b) $Ax = 0$ has only the trivial solution
(c) A is row equivalent to I
(d) All of the above

14. Suppose that y and z are both solutions to $Ax = 0$ then

(a) $Ax = 0$ has exactly two solutions
(b) $y = z$
(c) $y + z$ is a solution to $Ax = 0$
(d) None of the above

$x = y + z$
$Ax = Ay + Az$
$Ax = 0 + 0$

Question 3. (10%) Recall that the null space of an $m \times n$ matrix A is the set

$$N(A) = \{x \in R^n \mid Ax = 0\}.$$

Prove that $N(A)$ is a subspace of R^n.

$$\underset{m\times n}{A} = \begin{bmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ a_{21} & a_{22} \cdots & a_{2n} \\ \vdots & \ddots & \vdots \\ a_{m1} \cdots & a_{m2} & a_{mn} \end{bmatrix}$$

when $AX = 0$ is $N(A)$
then it has unique solution $N(A)\{x \in R^n \mid Ax = 0\}$

$$N(A) = \left[\begin{array}{ccc|c} a_{11} & a_{12} \cdots & a_{1n} & 0 \\ a_{21} & a_{22} \cdots & a_{2n} & 0 \\ \vdots & \ddots & \vdots & \vdots \\ a_{m1} \cdots & a_{n3} & a_{mn} & 0 \end{array}\right]$$

if we take y arbitrary elemen $\in R^n$

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$Ay = 0$

so if $A(x+y) = Ax + Ay$
$= 0$

and α is a scalar real number

$A(\alpha x) = \alpha A x$
$= 0$

Since $Ax = 0$

so $N(A)$ subspace in R^n

Question 4. (10%) Use Guauss-Jordan elimination to solve the system

$$\begin{array}{rcrcrcrcl} x_1 & + & 2x_2 & - & 3x_3 & + & x_4 & = & 1 \\ -x_1 & - & x_2 & + & 4x_3 & - & x_4 & = & 6 \\ -2x_1 & - & 4x_2 & + & 7x_3 & - & x_4 & = & 1 \end{array}$$

the equation < unknown
under determined
system
1) in consistent
2) consistent (infinity solution)

$$\left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 1 \\ -1 & -1 & 4 & -1 & 6 \\ -2 & -4 & 7 & -1 & 1 \end{array}\right] \begin{array}{l} \\ R_2+R_1 \\ R_3+2R_1 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & 1 \\ 0 & 1 & 1 & 0 & 7 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right] \begin{array}{l} R_1+3R_3 \\ R_2-R_3 \\ \end{array}$$

its Row echelon form

now i will use Jordan Gauss elimination to solve system

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 3 & 7 \\ 0 & \boxed{1} & 0 & -1 & 4 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right] R_1-2R_2$$

x_4 free

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 5 & -1 \\ 0 & 1 & 0 & -1 & 4 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right]$$

$\boxed{x_4 = \alpha}$

$x_3 + x_4 = 3 \Rightarrow \boxed{x_3 = 3 - \alpha}$

$x_2 - x_4 = 4 \Rightarrow \boxed{x_2 = 4 + \alpha}$

$x_1 + 5x_4 = -1 \Rightarrow \boxed{x_1 = -1 - 5\alpha}$

the Solution Set is $\{(-1-5\alpha), (4+\alpha), (3-\alpha), \alpha\}$

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Question 5.(8%) let A be an $n \times n$ nonsingular matrix, and suppose that $|A| = 2$. Find $|A^{-1} + \operatorname{adj}(A)|$.

$\cancel{|A| = 2}$

~~A is non singular~~

$\cancel{|A^{-1} + adj(A)|}$

$\cancel{adj(A) = |A| A^{-1}}$

$\cancel{A^{-1} = \frac{1}{|A|} adj(A)}$

$\cancel{adj(A) = A^{-1} |A|}$

$\cancel{A^{-1} + adj(A) = A^{-1} + A^{-1} |A|}$

$\cancel{AA^{-1} + A\,adj(A) = AA^{-1} + AA^{-1} |A|}$

$\cancel{|I + A\,adj(A)| = I + I|A|}$

$\cancel{1 + |A|\,|adj A| = 1 + |A|^n}$

$\cancel{adj(A) = \frac{|A|^n}{|A|}} \Rightarrow |adj\,A| = |A|^{n-1}$

$$\left| \frac{adj(A)}{|A|} + adj\,A\,|A| \right|$$

$$\left| \frac{3\,adj\,A}{2} \right|$$

$$\left(\frac{3}{2}\right)^n |adj\,A|$$

$A(adj\,A) = \det A\, I$

$\cancel{|A|\,|adj\,A| = 1}$

$|adj\,A| = \frac{1}{|A|}$

خلف الورقة $\longrightarrow$