MATH 234
MIDTERM EXAM

Student Name: Key II Student Number:

Instructor: Section:

Question 1. (22 points) Answer by true or false:

1. T A linear homogeneous system which has a nonzero solution must have infinitely many solutions.
2. T If S is a set of vectors that are linearly independent in a vector space V then any nonempty subset of S is linearly independent
3. F If S is a set of vectors that are linearly independent in a vector space V then any subset of V containing S is linearly independent
4. T $\text{Span}\{1+x, 1-x\}$ is a subspace of P_2
5. T Any nonsingular matrix can be written as a product of elementary matrices.
6. F If A is row equivalent to B then both A and B are nonsingular.
7. T If $\text{span}\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\} = R^3$ then $\text{span}\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}\} = R^3$ for any $\mathbf{x} \in R^3$.
8. T If A is singular then $\text{adj}(A)$ is singular.
9. T If A is singular and U is the row echelon form of A, then $\det(U) = 0$.
10. F Suppose that $\{f_1, f_2, \ldots, f_n\} \subseteq C^{n-1}[a, b]$. If $W[f_1, \ldots, f_n] = 0$, where W denotes the Wronskien, then $f_1, f_2, \ldots, f_n$ are linearly dependent.
11. F If A is row equivalent to B, then $A\mathbf{x} = \mathbf{b}$ and $B\mathbf{x} = \mathbf{b}$ have the same solution set.
12. F If a system of linear equations is overdetermined and consistent, then it must have infinitely many solutions.
13. F If A, B, C are $n \times n$ matrices such that $AB = AC$, then $B = C$.
14. F Every linear system can be solved using Cramer's Rule.
15. F If E_1 and E_2 are elementary $n \times n$ matrices, then E_1E_2 is elementary.
16. T $(AB)C = A(BC)$ for all matrices A, B, and C when multiplication is allowed.
17. T If A and B are $n \times n$ matrices and A is singular, then AB is singular.
18. F If A is a singular $n \times n$ matrix, then $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions for every vector $\mathbf{b} \in R^n$.
19. T The set $\{1, \sin^2 x, \cos^2 x\}$ is linearly dependent in $C[0, \pi]$.
20. T If A, B are $n \times n$ matrices, $|A| = 2$ and $|B| = -2$, then $|A^{-1}B^T| = -1$.
21. F If A and B are symmetric $n \times n$ matrices then AB is symmetric.
22. F If S is a subspace of V then any set of vectors in S that spans S also spans V.

Question 2 (36 points) Circle the most correct answer:

1. One of the following is not a subspace of P_3

   (a) $\{p \in P_3 \,|p(2) = p(5)\}$
   (b) $\{p \in P_3 \,|p(0) = 2\}$
   (c) $\{p \in P_3 \,|p(1) = p(-1)\}$
   (d) $\{p \in P_3 \,|p(2) = 0\}$

2. One of the following sets is linearly independent in P_3

   (a) $\{1 + x, 1 - x, 1\}$
   (b) $\{2x, 2 - x, x^2\}$
   (c) $\{x, x^2, 2x + 3x^2\}$
   (d) $\{2, 2 - x, x\}$

3. One of the following sets is a subspace of $C[-1, 1]$

   (a) $\{f(x) \in C[-1, 1] \;;\; f(1) = 1\}$
   (b) $\{f(x) \in C[-1, 1] \;;\; f(1) = -1\}$
   (c) $\{f(x) \in C[-1, 1] \;;\; f(1) = 0 \text{ or } f(-1) = 0\}$
   (d) $\{f(x) \in C[-1, 1] \;;\; f(1) = 0\}$

4. Suppose A and B are $n \times n$ nonsingular matrices. Then

   (a) ABA^{-1} is nonsingular
   (b) AB is nonsingular
   (c) B^TA^{-1} is nonsingular
   (d) all of the above

5. Let $A = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$. Then the adjoint of A is

   (a) $\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}$

   (b) $\begin{bmatrix} 2 & -1 \\ -3 & 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 2 & 1 \\ -3 & 1 \end{bmatrix}$

   (d) $\begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$

6. consider the linear system $\begin{array}{ccc} 4x_1 & +kx_2 & = 4 \\ kx_1 & +x_2 & = -2 \end{array}$.

   (a) The system has a unique solution if

      i. $k \neq -2, 2$

      ii. $k = 2$

      iii. $k \neq -2$

      iv. none of the above

   (b) The system has infinitely many solutions if

      i. $k = -2$

      ii. $k = 2$

      iii. $k \neq 2$

      iv. none of the above

   (c) The system is inconsistent if

      i. $k \neq -2$

      ii. $k = 2$

      iii. $k = -2$

      iv. none of the above

7. The vectors $1, x, x^2, x^2 + x - 1$

   (a) span P_3

   (b) span P_4

   (c) are linearly independent in P_3

   (d) are linearly independent in P_4

8. Let $A = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$ then the $(2, 3)$ entry of A^2 is

   (a) 0

   (b) 1

   (c) 2

   (d) 3

9. One of the following matrices is in reduced row echelon form

(a) $\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

10. One of the following is a spanning set of R^3

(a) $\{(2,0,0)^T, (0,2,0)^T, (1,1,0)^T, (0,-3,0)^T\}$
(b) $\{(-1,-1,-1)^T, (-1,-2,-1)^T, (-1,0,0)^T\}$
(c) $\{(1,-1,1)^T, (-4,4,-4)^T, (3,0,5)^T\}$
(d) $\{(2,0,0)^T, (0,3,-4)^T\}$

11. One of the following is a subspace of $R^{n\times n}$

(a) All nonsingular $n \times n$ matrices
(b) All triangular $n \times n$ matrices
(c) All singular $n \times n$ matrices
(d) All upper triangular $n \times n$ matrices

12. The set of vectors $\{(1,a)^T, (b,1)^T\}$ is a spanning set for R^2 if

(a) $ab \neq 1$
(b) $a \neq 1$ and $b \neq 1$
(c) $ab = 1$
(d) $a \neq b$

13. The set $\{(1,1,1)^T, (1,1,c)^T, (1,c,1)^T\}$ is linearly independent in R^3 if

(a) $c = -1$
(b) $c = 1$
(c) $c \neq 1$
(d) c is any real number

14. Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $B = \begin{bmatrix} a & 4d & a+g \\ b & 4e & b+h \\ c & 4f & c+i \end{bmatrix}$. If $|A| = 3$ then $|B| =$

(a) 3
(b) 12
(c) 6
(d) 24

15. If A is a 3×3 matrix with $\det(A) = 2$ then $\det(\text{adj}(A))$ is

(a) 4
(b) 0
(c) 2
(d) $\frac{1}{2}$

16. Suppose that $\mathbf{y}$ and $\mathbf{z}$ are both solutions to $A\mathbf{x} = \mathbf{0}$ then

(a) $\mathbf{y} = \mathbf{z}$
(b) $\mathbf{y} + \mathbf{z}$ is a solution to $A\mathbf{x} = \mathbf{0}$
(c) $A\mathbf{x} = \mathbf{0}$ has exactly two solutions
(d) None of the above