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Birzeit University
Mathematics Dept.
Math. 234-a

M. Saleh

Midterm Exam — Summer Semester 2014/2015

Student Name: Basil AlKhateeb Number: 1142068 Section ___

Q1 (40 points) Answer the following statements by true or false:

(1) Any subset of a vector space that does not contain the zero vector is not a subspace. True

(2) If $v_1, v_2, \ldots, v_n$ span a vector space V and v_1 is a linear combination of $v_2, \ldots, v_n$, then $V = Span\{v_2, \ldots, v_n\}$. True

(3) If two none zero vectors in a vector space V are linearly dependent, then each one of them is a scalar multiple of the other. True

(4) The vectors $(4,2,3)^T, (2,3,1)^T, (2,-5,3)^T, (2,-1,3)^T$ are linearly dependent. True

(5) If 6 vectors span a vector space V, then a collection of 14 vectors in V is linearly dependent. True

(6) If V is a vector space with dimension $n > 0$, then any set of $m < n$ vectors in V does not span V. True

(7) If V is a vector space with dimension $n > 0$, then any set of $m > n$ vectors in V are linearly dependent. True

(8) If S is a subset of a vector space V that contains the zero vector, then S is a subspace of V. False

(9) The set $S = \{v_1, \ldots, v_n\}$ is a spanning set of a vector space V if every vector in V is a linear combination of the set S. True

(10) If two vectors in a vector space V are linearly dependent, then one of them is a scalar multiple of the other. True

(11) If 3 vectors span a vector space V, then a collection of 6 vectors in V span V. False

(12) The coordinate vector of $2 + 6x$ with respect to the basis $[2x, 4]$ is $(2,3)^t$ False

(13) If two matrices are row equivalent, they must have the same null space. True

(14) If A is an invertible matrix then A^t is invertible. True

(33) If two matrices are row equivalent, they must have the same determinant. False

(34) If S is a subset of a vector space V that contains the zero vector, then S is a subspace of V. False

(35) The set $S = \{v_1, ..., v_n\}$ is a spanning set of a vector space V if every vector in V is a linear combination of the set S. True

(36) $S = \{f \in C(R) : f(0) = 0\}$ is not a subspace of $V = C(R)$ False

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(37) $S = \{A \in R^{2\times2} : a_{11} = 0\}$ is a subspace of $V = R^{2\times2}$ True

(38) $S = \{v = (x, y) \in R^2 : x + y = 1\}$ is not a subspace of $V = R^2$ True

(39) A basis for the subspace $S = \{(a + b + 2c, a + 2b + 4c, b + 2c)^T, a, b, c \in R\}$ is $\{(1, 1, 0)^T, (1, 2, 1)^T, (1, 2, 1)^T\}$ False

(40) If $(A|b) = \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ -1 & 1 & -1 & 0 \\ -1 & 0 & a & b \end{array}\right)$ is the augmented matrix of the system $AX = b$ then the system has no solution if $a = -2$ False

$\& \; b=-1$

$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2+a & b+1 \end{bmatrix}$

Q2 :(26 points) Choose the correct answer.

(1) An $n \times n$ matrix A is invertible if $A A^{-1} = I$

(a) there exists a matrix B such that $AB = I$

(b) The columns of A are li

(c) The rows of A are li

(d) $N(A) = \{0\}$

(e) all

(2) Let S be a finite subset of a subspace W of R^n. Then S is a basis for W if

(a) S is linearly independent

(b) S spans W

(c) the number of vectors in S equals the dimension of W

(d) every vector in W is a linear combination of vectors in S

(e) None

(3) Suppose that W is a subspace of R^n. Then

(a) the dimension of W is greater than n

(b) every basis of R^n contains a basis of W

(c) every linearly independent subset of W has at most n vectors

(d) the dimension of W equals n

(33) If two matrices are row equivalent, they must have the same determinant. False

(34) If S is a subset of a vector space V that contains the zero vector, then S is a subspace of V. False

(35) The set $S = \{v_1, ..., v_n\}$ is a spanning set of a vector space V if every vector in V is a linear combination of the set S. True

(36) $S = \{f \in C(R) : f(0) = 0\}$ is not a subspace of $V = C(R)$ False

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(37) $S = \{A \in R^{2\times 2} : a_{11} = 0\}$ is a subspace of $V = R^{2\times 2}$ True

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(39) A basis for the subspace $S = \{(a+b+2c, a+2b+4c, b+2c)^T, a, b, c \in R\}$ is $\{(1,1,0)^T, (1,2,1)^T, (1,2,1)^T\}$ False

(40) If $(A|b) = \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ -1 & 1 & -1 & 0 \\ -1 & 0 & a & b \end{array}\right)$ is the augmented matrix of the system $AX = b$ then the system has no solution if $a = -2$ False

& $b = -1$

Q2 :(26 points) Choose the correct answer.

(1) An $n \times n$ matrix A is invertible if $A A^{-1} = I$

(a) there exists a matrix B such that $AB = I$

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(d) $N(A) = \{0\}$

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(2) Let S be a finite subset of a subspace W of R^n. Then S is a basis for W if

(a) S is linearly independent

(b) S spans W

(c) the number of vectors in S equals the dimension of W

(d) every vector in W is a linear combination of vectors in S

(e) None

(3) Suppose that W is a subspace of R^n. Then

(a) the dimension of W is greater than n

(b) every basis of R^n contains a basis of W

(c) every linearly independent subset of W has at most n vectors

(d) the dimension of W equals n

(e) None

(9) For any vector space V,

(a) If V is finite-dimensional, then V is a subspace of R^n for some positive integer n

(b) If V is infinite-dimensional, then every infinite subset of V is linearly independent

(c) If V is a nonempty subset of a vector space W, and the operations defined on V coincide with the operations defined on W, then V is a subspace of W

(d) If V is finite-dimensional, then no infinite subset of V is linearly independent

~~(e) None~~

(10) The dimension of the subspace $S = \{(a+b+2c, a+2b+4c, b+2c)^T, a, b, c \in R\}$ is

$$\begin{bmatrix} a+b+2c \\ a+2b+4c \\ b+2c \end{bmatrix}$$

$$a\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + b\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + c\begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$$

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4

(11) A basis for the vector space spanned by $1-x-x^2, 1+x+x^2, 2-x, 1-x$ from this set of vectors is

$\alpha - \alpha x - \alpha x^2 + \beta - \beta x = 0$

$-\alpha x^2 - (\alpha+\beta)x$

$\alpha = 0$

$\beta = 0$

(a) $1-x-x^2, 1+x+x^2, 2-x$

(b) $1-x-x^2, 1+x+x^2$

(c) $1-x-x^2, 1+x+x^2, 2-x, 1-x$

(d) $1-x-x^2, 1-x$

(e) $1-x-x^2, 2-x$

(12) The dimension of the null space of $\begin{pmatrix} 1 & 1 & 2 & 1 & 4 \\ 2 & -1 & 2 & -1 & 6 \\ 3 & 0 & 4 & 0 & 10 \end{pmatrix}$ is

$R_2 - 2R_1$, $R_3 - 3R_1$

$$\begin{pmatrix} 1 & 1 & 2 & 1 & 4 \\ 0 & -3 & -2 & -3 & -2 \\ 0 & -3 & -2 & -3 & -2 \end{pmatrix}$$

$$a\begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + b\begin{pmatrix} -2 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + c\begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} -4d-c-2b-a \\ a \\ b \\ c \\ d \end{pmatrix}$$

$$+\, d\begin{pmatrix} -4 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

(a) 0

(b) 1

(c) 2

(d) 3

~~(e) 4~~

$$\begin{pmatrix} 2c+3d+2b \\ b \\ d \\ c \end{pmatrix}$$

(13) One of the following set of vectors are linearly independent

(a) $(1,1,2,1,4), (2,2,4,2,8)$

(b) $(1,1,2,1,4), (2,-1,2,-1,6), (0,0,0,0,0)$

(c) $x, 1, x^2+1$

(d) $(1,2,3), (0,1,0), (0,0,1), (1,1,1)$

Q3 (40 points):(a) If U, W are subspaces of a vector space V. Show that $U \cap W$ is a subspace of V

① since U, W are subspaces $0 \in U$, $0 \in W$

$\Rightarrow$ $0 \in U \cap W$ $\therefore$ $U \cap W$ is not empty

② let A, B be vectors and $A, B \in U \cap W \Rightarrow A, B \in U$, $A, B \in W$

$\Rightarrow$ $A+B \in U$, $A+B \in W$ (since they are subspaces)

$\rightarrow$ $A+B \in U \cap W$ ✓

③ let $A \in U \cap W$, $\alpha \in \mathbb{R}$ $\Rightarrow A \in U, A \in W$

$\alpha A \in U$, $\alpha A \in W$ (since U, W are subspaces)

$\Rightarrow$ $\alpha A \in U \cap W$

$\therefore$ $U \cap W$ satisfies the conditions and is a subspace ⊠

(b) If U, W are subspaces of a vector space V. Show that $U \cup W$ need not be a subspace of V

let A, B be vectors : $A \in U$, $B \in W$

$A, B \in (U) \cup (W)$, but there sum $A+B$ might not be in $(U) \cup (W)$

(c) If V be a vector space and let t be a nonzero real constant and v a vector in V such that $tv = 0$. Show that $v = 0$

$tv = 0$ since t is non zero

$v = \frac{1}{t} 0$

so if we exclude 1 element a from the subspace a
elements will be excluded exept zero

$\Rightarrow W = \{0\}$ or $W = \mathbb{R}$, since W is nonz

$\Rightarrow W = \mathbb{R}$ ▨

(e) Let $S = \{(0, a)^t : a \in R\}$. Show that S is a subspace of R^2.

(f) Let A, B be symmetric matrices, and $AB = BA$. Show that AB is symmetric

(e) $S = \left\{ \begin{pmatrix} 0 \\ a \end{pmatrix} : a \in \mathbb{R} \right\}$

① $\begin{pmatrix} 0 \\ 0 \end{pmatrix} \in S$ taking $a = 0$, so the zero vector $\in$
and S is not empty

② let $\begin{pmatrix} 0 \\ a \end{pmatrix}, \begin{pmatrix} 0 \\ b \end{pmatrix} \in S$ $(a, b \in \mathbb{R})$ $\begin{pmatrix} 0 \\ a \end{pmatrix} + \begin{pmatrix} 0 \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ a+b \end{pmatrix}$

since $a+b \in \mathbb{R} \Rightarrow \begin{pmatrix} 0 \\ a+b \end{pmatrix} \in S \Rightarrow$ the sum of any
2 vectors in $S \in S$

③ let $\begin{pmatrix} 0 \\ a \end{pmatrix} \in S$, $\alpha \in \mathbb{R}$ $\alpha \begin{pmatrix} 0 \\ a \end{pmatrix} = \begin{pmatrix} 0 \\ \alpha a \end{pmatrix}$

since $\alpha a \in \mathbb{R} \Rightarrow \begin{pmatrix} 0 \\ \alpha a \end{pmatrix} \in S$

(f) AB is symmetric if $(AB)^t = AB$

$(AB)^T = B^T A^T$ $\quad (A = A^T, B = B^T)$

$(AB)^T = BA$ $\quad (BA = AB)$

$(AB)^T = AB \Rightarrow AB$ is symmetric ▨

(g) Let A be a square $n \times n$ nonsingular matrix. Show that $|(adj(A))| = |A|^{n-1}$

$$A^{-1} = \frac{1}{|A|} \, adj(A)$$

taking the determinant of both sides $\left(\left| \frac{1}{|A|} adj(A) \right| = \left(\frac{1}{|A|}\right)^n |adj(A)| \right)$

$$|A^{-1}| = \left(\frac{1}{|A|}\right)^n |adj(A)|$$

since $|A^{-1}| = \frac{1}{|A|}$

$$\frac{1}{|A|} = \frac{1}{(|A|)^n} |adj(A)|$$

$$\Rightarrow |adj(A)| = \frac{(|A|)^n}{|A|}$$

$$\therefore |adj(A)| = |A|^{n-1} \quad \square$$

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(h) Let the system $Ax = 0$ have a nonzero solution. If the system $Ax = b$ is consistent, prove that $Ax = b$ has infinitely many solutions

since $Ax = 0$ have a non zero solution, A is singular
so the system $Ax = b$ with A singular has either no solution or infinitly many solutions
& $Ax = b$ is consistent $\Rightarrow$ $Ax = b$ has infinitly many solutions $\square$

also since A contains a free variable (since $Ax = 0$ has infinite solutions) $Ax = b$ will have infinite or no solutions, since it is consistent it has infinite solutions $\square$.