Birzeit University
Mathematics Department

Extra 4.1 Math 234 2018/2019

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(Q1) Let $L : P_3 \longrightarrow \mathbb{R}^2$ be a linear transformation defined by

$$L(ax^2 + bx + c) = \begin{pmatrix} a+b \\ a-c \end{pmatrix}$$

(a) Find a basis and dimension of $ker(L)$.

(b) Find a basis and the dimension of $range(L)$.

(c) Is L one-to one?

(d) Is L onto?

(e) If $S = span(x^2+1)$, find the image of S.

(Q2) Let $L : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ be a linear transformation defined by $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x-y+z \\ 2x-4z \end{bmatrix}$

(a) Find a basis and dimension of $ker(L)$.
(b) Find a basis and dimension of $range(L)$.

(Q3) Let $L : \mathbb{R}^3 \longrightarrow P_4$ given by $L\left((a,b,c)^T\right) = (a+b)x^3 + (b+c)x^2 + (a+c)x$.

(a) Find a basis and dimension of $ker(L)$.
(b) Find a basis and dimension of $range(L)$.

(Q4) Let $L : P_3 \longrightarrow P_3$ be the linear transformation defined by $L(p(x)) = x^2p''(x) + p'(x) + p(0)$

(a) Find a basis and dimension of $ker(L)$.
(b) Find a basis and dimension of $range(L)$.

(Q5) Let $L : P_3 \longrightarrow \mathbb{R}^{2\times 2}$ be a linear transformation defined by $L(ax^2+bx+c) = \begin{pmatrix} a+b & a \\ a-b-c & b+c \end{pmatrix}$.

(a) Find a basis and dimension of $ker(L)$.
(b) Find a basis and dimension of $range(L)$.

(Q6) Let $L : P_3 \longrightarrow \mathbb{R}^2$ be defined by $L(p(x)) = \begin{pmatrix} \int_0^1 p(x)dx \\ p(0) \end{pmatrix}$.

(a) Find a basis and dimension of $ker(L)$.
(b) Find a basis and dimension of $range(L)$.

(Q7) Let $L : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ be a linear transformation defined by $L((a,b,c)^T) = (a+b+c, 0)^T$.

(a) Find a basis and dimension of $ker(L)$.
(b) Find a basis and dimension of $range(L)$.

(Q8) Let $L : P_3 \longrightarrow \mathbb{R}^2$ be a linear transformation such that $L(p(x)) = \begin{pmatrix} p''(x) - p'(1) \\ p(0) \end{pmatrix}$.

(a) Find $ker(L)$ and its dimension.

(b) Find range(L) and its dimension.

(c) Is L one-to-one? Onto? Why?

(d) Let $S = P_1$. Find $L(S)$

Short Answers

(Q1)

(a) Basis $= \{x^2 - x + 1\}$ $\quad dim(ker(L)) = 1$

(b) Basis = Any basis of $\mathbb{R}^2$ $\quad dim(range(L)) = 2$

(c) No.

(d) Yes.

(e) $span(e_1)$

(Q2)

(a) Basis $= \{(2,3,1)^T\}$ $\quad dim(ker(L)) = 1$

(b) Basis = Any basis of $\mathbb{R}^2$ $\quad dim(range(L)) = 2$

(Q3)

(a) Basis $= \{(0,0,0)^T\}$ $\quad dim(ker(L)) = 0$

(b) Basis $= \{x^3 + x, x^3 + x^2, x^2 + x\}$ $\quad dim(range(L)) = 3$

(Q4)

(a) Basis $= \{1 - x\}$ $\quad dim(ker(L)) = 1$

(b) Basis $= \{x^2 + x, 1\}$ $\quad dim(range(L)) = 2$

(Q5)

(a) Basis $= \{0\}$ $\quad dim(ker(L)) = 0$

(b) Basis $= \left\{ \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ -1 & 1 \end{pmatrix} \right\}$ $\quad dim(range(L)) = 3$

(Q6)

(a) Basis $= \{2x - 3x^2\}$ $\quad dim(ker(L)) = 1$

(b) Basis = Any basis of $\mathbb{R}^2$ $\quad dim(range(L)) = 2$

(Q7)

(a) Basis $= \{(1, 0, -1)^T, (0, 1, -1)^T\}$ $\quad dim(ker(L)) = 2$

(b) Basis $= \{e_1\}$ $\quad dim(range(L)) = 1$

(Q8)

(a) Basis $= \{x^2\}$ $\quad dim(ker(L)) = 1$

(b) Basis = Any basis of $\mathbb{R}^2$ $\quad dim(range(L)) = 2$

(c) Onto but not one-to-one.

(d) $span(e_2)$