Birzeit University
Mathematics Department
Math234
Quiz#3

Instructor: Dr. Ala Talahmeh | **First Semester 2021/2022**
Name:.. | **Number:..............**
Time: 15 minutes | **Date: 30/11/2021**

Exercise#1 [**5 marks**]. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}$ be vectors in the vector space $\mathbb{R}^{2\times 2}$. Determine whether if A, B, C are **linearly dependent or linearly independent**.

Solution. Let $a\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix} + c\begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$\Rightarrow \begin{cases} a + 3c = 0 \\ 2a + b = 0 \\ 3b + 2c = 0 \\ 3a + 2b + c = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 3 & 2 & 0 \\ 3 & 2 & 1 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & -6 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & 2 & -8 & 0 \end{array}\right] \begin{array}{l} -2R_1 + R_2 \\ \\ -3R_1 + R_4 \end{array}$$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & -6 & 0 \\ 0 & 0 & 20 & 0 \\ 0 & 0 & 4 & 0 \end{array}\right] \begin{array}{l} -3R_2 + R_3 \\ -2R_2 + R_4 \end{array}$$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & -6 & 0 \\ 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] -\tfrac{1}{5}R_3 + R_4$$

the equivalent system is
$$\begin{aligned} a + 3c &= 0 \\ b - 6c &= 0 \\ 20c &= 0 \end{aligned}$$

$\Rightarrow a = b = c = 0 \Rightarrow$ lin. indep.

Exercise#2 [5 marks]. Let S be a subset of $\mathbb{R}^3$ such that $S = \left\{(x,y,z)^T : x^2 + (y+z)^2 = 0\right\}$. Show that S is a subspace of $\mathbb{R}^3$.

Sol. $x^2 + (y+z)^2 = 0 \Rightarrow \boxed{x = 0}$ and $y + z = 0$

$\boxed{y = -z}$

$\therefore S = \left\{(x,y,z)^T : (0,-z,z) : z \in \mathbb{R}\right\}$.

(i) $(0,0,0)^T \in S \Rightarrow S \neq \phi$.

(ii) let $(0,-a,a)^T, (0,-b,b)^T \in S$. Then

$(0,-a,a)^T + (0,-b,b)^T = (0, -(a+b), a+b) \in S$.

(iii) let $\alpha \in \mathbb{R}$, $(0,-z,z) \in S$. Then,

$\alpha(0,-z,z) = (0, -\alpha z, \alpha z) \in S$.

$\therefore S$ is a subspace of $\mathbb{R}^3$. $\square$