Birzeit University
Mathematics Department
Math234
Quiz 2

Instructor: Dr. Ala Talahmeh — First Semester 2021/2022
Name:... — Number:..............
Section: 5 — Date: 28/10/2021

Exercise 1 [2.5 marks]. Find a 2×2 singular matrix A and a 2×2 nonsingular matrix B such $A+B$ is singular.

For example $A=\begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$, $B=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Exercise 2 [2.5 marks]. Let

$$A=\begin{bmatrix} \alpha & \beta \\ 0 & \alpha \end{bmatrix}$$

where α and β are real numbers. Compute A^n, where n is a positive integer.

Sol. $A^2=\begin{bmatrix} \alpha & \beta \\ 0 & \alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ 0 & \alpha \end{bmatrix}=\begin{bmatrix} \alpha^2 & 2\alpha\beta \\ 0 & \alpha^2 \end{bmatrix}$

$A^3=A^2A=\begin{bmatrix} \alpha^2 & 2\alpha\beta \\ 0 & \alpha^2 \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ 0 & \alpha \end{bmatrix}$

$=\begin{bmatrix} \alpha^3 & 3\alpha^2\beta \\ 0 & \alpha^3 \end{bmatrix}$

$\vdots$

$A^n=\begin{bmatrix} \alpha^n & n\alpha^{n-1}\beta \\ 0 & \alpha^n \end{bmatrix}$, $n=1,2,\dots$

Exercise 3 [5 marks].

(a) Let A be a square matrix matrix such that $A^2-3A+I=0$. Show that A is nonsingular and find A^{-1} (as a function of A).

Since $A^2-3A+I=0$, then $A(3I-A)=(3I-A)A=I$

$\therefore$ A is nonsingular and $A^{-1}=3I-A$.

(b) **Prove or disprove.**
Let A be a square matrix. If $A^2 = I$, then $A = I$ or $A = -I$.

Ans. False

Counterexample. $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

but A is neither I nor $-I$.