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Birzeit University
Mathematics Department
Math234
Quiz 1

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Name:.....
Section: 2

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Question I [10 points]. Consider the linear system

$$\begin{aligned} x + y + 2\alpha z &= -1 \\ 2x + y + (\alpha - 1)z &= \beta \\ -x - y + (\alpha^2 + 1)z &= \alpha - \beta. \end{aligned}$$

- Under what conditions on the constants α and β does the the system have:
 - no solutions,
 - a unique solution,
 - infinitely many solutions.
- Find the solution set in the case when the system has infinitely many solutions.

1.
$$\left[\begin{array}{ccc|c} 1 & 1 & 2\alpha & -1 \\ 2 & 1 & \alpha-1 & \beta \\ -1 & -1 & \alpha^2+1 & \alpha-\beta \end{array} \right] \text{ (1 pt)}$$

$$\begin{array}{l} \rightarrow \\ -2R_1 + R_2 \\ R_1 + R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 2\alpha & -1 \\ 0 & -1 & -3\alpha-1 & \beta+2 \\ 0 & 0 & (\alpha+1)^2 & \alpha-\beta-1 \end{array} \right] \text{ (2 pts)}$$

(a) ~~no~~ No solution
 (1 pt) if $\alpha = -1, \alpha - \beta - 1 \neq 0$
 i.e, $\alpha = -1, \beta \neq -2$

(b) unique if $\alpha \neq -1, \beta \in \mathbb{R}$

Good Luck

(c) Infinite if $\alpha = -1, \beta = -2$

3 pts

2.

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & -1 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\xrightarrow{-R_2} \left[\begin{array}{ccc|c} 1 & 1 & -2 & -1 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

x_1, x_2 leading, $x_3 = t$ free

$$\begin{cases} x_1 + x_2 - 2x_3 = -1 \\ x_2 - 2x_3 = 0 \Rightarrow x_2 = 2t \\ x_1 + 2t - 2t = -1 \Rightarrow x_1 = -1 \end{cases}$$

\therefore the solution set = $\{ (-1, 2t, t) : t \in \mathbb{R} \}$.