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Birzeit University
Mathematics Department
Math234
Quiz 1

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Name:..

Section: 5

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Question I [10 points]. Consider the linear system

$$\begin{aligned} x - y - \alpha z &= 3 \\ -x + 2y - 2z &= 2\beta \\ 2x - y + \beta z &= \beta + 1. \end{aligned}$$

1. Under what conditions on the constants α and β does the the system have:

   (a) no solutions,

   (b) a unique solution,

   (c) infinitely many solutions.

2. Find the solution set in the case when the system has infinitely many solutions.

1.
$$\begin{bmatrix} 1 & -1 & -\alpha & | & 3 \\ -1 & 2 & -2 & | & 2\beta \\ 2 & -1 & \beta & | & \beta+1 \end{bmatrix}$$ (1 pts)

$\xrightarrow{\begin{array}{l} R_1 + R_2 \\ -2R_1 + R_3 \end{array}}$
$$\begin{bmatrix} 1 & -1 & -\alpha & | & 3 \\ 0 & 1 & -2-\alpha & | & 2\beta+3 \\ 0 & 1 & 2\alpha+\beta & | & \beta-5 \end{bmatrix}$$ (1)

$\xrightarrow{\begin{array}{l} \\ -R_2 + R_3 \end{array}}$
$$\begin{bmatrix} 1 & -1 & -\alpha & | & 3 \\ 0 & 1 & -2-\alpha & | & 2\beta+3 \\ 0 & 0 & 3\alpha+\beta+2 & | & -\beta-8 \end{bmatrix}$$ (1)

Good Luck

(1) (a) no solution if $3\alpha + \beta + 2 = 0$ and $\beta \neq -8$

(1) (b) unique if $3\alpha + \beta + 2 \neq 0$

(2) (c) infinite if $3\alpha + \beta + 2 = 0$ and $-\beta - 8 = 0$

i.e, $\boxed{\alpha = 2}$ and $\boxed{\beta = -8}$

2. $\begin{bmatrix} 1 & -1 & -2 & | & 3 \\ 0 & 1 & -4 & | & -13 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$

(3)

x_1, x_2 leading $\boxed{x_3 = t}$ free

$\boxed{x_2 = 4x_3 - 13 = 4t - 13}$

$x_1 = x_2 + 2x_3 + 3$

$= 4t - 13 + 2t + 3$

$\boxed{x_1 = 6t - 10}$

$S.S = \{ (6t - 10, 4t - 13, t) : t \in \mathbb{R} \}.$