Key

50

Birzeit University
Mathematics Department

Quiz # 5 + 6 + 7 | Math 234 | 2018/2019

Name................................ Number................. Section ....

(Q1) [*15 points*] Answer by True (T) or False (F)

(1) ..F... If V is a vector space with $dim(V) = 4$ and $v_1, v_2, v_3, v_4 \in V$, then span $(v_1, v_2, v_3, v_4) = V$

(2) ..F... Let V be a vector space with $dim(V) = n$ and let S be a subspace of V, then $0 < dim(S) < n$

(3) ..F... If V is an infinite-dimensional vector space, then any subspace of V is infinite-dimensional.

(4) ...F. Every linearly independent set of vectors in P_n must contain n polynomials.

(5) ...T.. If V is a vector space with $dim(V) = n$, then any $n + 1$ vectors in V are linearly dependent.

(6) ...F.. If $v_1, v_2, \cdots, v_n$ are linearly independent vectors in V, then V is finite dimensional.

(7) ....T.. If x_1, x_2 are linearly independent vectors in $\mathbb{R}^3$, then $\exists\ x \in \mathbb{R}^3$ such that $\text{span}(x_1, x_2, x) = \mathbb{R}^3$

(8) ....T.. The vectors e^x, xe^x, x are linearly independent in $C[0, 1]$

(9) ..F... Let $f, g, h \in C^2[a, b]$. If $W[f, g, h](x) = 0\ \forall\ x \in [a, b]$, then f, g, h are linearly dependent in $C[a, b]$

(10) ..T... $\text{span}(1 + x, 1 - x)$ is a subspace of P_2

(11) ....T.. If $\text{span}(x_1, x_2, x_3) = \mathbb{R}^3$, then $\text{span}(x_1, x_2, x_3, x) = \mathbb{R}^3$ for any $x \in \mathbb{R}^3$

(12) ..F... If S is a subspace of V, then any set of vectors in S that spans S also spans V

(13) ..F... If S is a set of linearly independent vectors in V, then any subset of V containing S is also linearly independent.

(14) ....T.. If S is a set of linearly independent vectors in V, then any nonempty subset of S is also linearly independent.

(14) ....T.. If S is a subspace of a vector space V, then S is also a vector space.

(15) ....T.. If $\{x_1, x_2, x_3\}$ is a subset of a vector space V and $\text{span}(x_1, x_2) = \text{span}(x_1, x_2, x_3)$, then x_1, x_2, x_3 are linearly dependent in V.

(16) ...F.. The sum of two triangular matrices is triangular.

(17) ....T.. If S and T are subspaces of a vector space V, then $S \cap T$ is a subspace of V

15

(Q2) [*20 points*] Circle the correct answer.

20

(1) One of the following is not a basis of P_3

(a) $\{1, 2x, x^2 - x\}$

(b) $\{x - 1, x^2 + 1, x^2 - 1\}$

(c) $\{x, x^2 + 3, x^2 - 5\}$

(d) $\{x^2 + 1, x^2 - 1, 2\}$

(2) If V is a vector space with $dim(V) = n$, then

(a) Any n linearly independent vectors in V span V

(b) Any spanning set of V must contain at most n vectors

(c) Any set in V containing less than n vectors must be linearly independent.

(d) All of the above.

(3) $\dim(\text{span}(1 - x, x^2, 3 + x^2, 1 + x^2)) =$

(a) 0

(b) 1

(c) 2

(d) 3

(4) One of the following sets is linearly independent in P_3

(a) $\{2, 2 - x, x\}$

(b) $\{2x, 2 - x, x^2\}$

(c) $\{1 + x, 1 - x, 1\}$

(d) $\{x, x^2, 2x + 3x^2\}$

(5) Suppose that a vector space V contains n linearly independent vectors, then

(a) Any n vectors in V are linearly independent.

(b) Any set in V containing more than n vectors is linearly dependent.

(c) If S is a set spanning V, then it must contain at least n vectors.

(d) If S is a set spanning V, then it must contain at most n vectors.

(6) Suppose that the set $\{v_1, v_2, v_3\}$ is linearly independent in a vector space V, then

(a) The set $\{v_1, v_1 + v_2, v_2 + v_3\}$ is linearly independent in V

(b) The set $\{v_1 + v_2, v_1 + v_3, v_2 + v_3\}$ is linearly independent in V

(c) The set $\{v_1, v_2, v_1 + v_2 + v_3\}$ is linearly independent in V

(d) All of the above.

(7) One of the following is a subspace of $\mathbb{R}^{n\times n}$

(a) All triangular $n\times n$ matrices.

(b) All singular $n\times n$ matrices.

(c) All upper triangular $n\times n$ matrices.

(d) All nonsingular $n\times n$ matrices.

(8) One of the following is not a subspace of P_3

(a) $\{p(x)\in P_3 \mid p(2)=0\}$

(b) $\{p(x)\in P_3 \mid p(1)=p(-1)\}$

(c) $\{p(x)\in P_3 \mid p(0)=2\}$

(d) $\{p(x)\in P_3 \mid p(2)=p(5)\}$

(9) Let $S=\{p(x)\in P_3 \mid p(0)=0 \text{ and } p(1)=0\}$. One of the following is a basis of S

(a) $\{1,x,x^2\}$

(b) $\{x,x^2\}$

(c) $\{x^2-1\}$

(d) $\{x^2-x\}$

(10) The set $\{(1,1,1)^T,(1,1,c)^T,(1,c,1)^T\}$ is a basis of $\mathbb{R}^3$ if

(a) $c\neq 1$ and $c\neq -1$

(b) $c\neq 1$

(c) $c\neq -1$

(d) $c=1$ or $c=-1$

(Q3) [*5 points*] Let A be an $m\times n$ matrix. Show that the null space of A is a subspace of $\mathbb{R}^n$

proof $N(A)=\{x\in\mathbb{R}^n \mid Ax=0\}$

(i) $0\in N(A)$ because $A0=0$ $\Rightarrow N(A)\neq\phi$ 1

(ii) If $x,y\in N(A)$, then $Ax=0$ & $Ay=0$ 2

and so $A(x+y)=Ax+Ay=0+0=0$

$\Rightarrow$ ~~N(A)~~

$\Rightarrow N(A)$ is closed under $+$

(iii) If $x\in N(A)$ & $\alpha\in\mathbb{R}$ then $Ax=0$ 2

& so $A(\alpha x)=\alpha Ax=\alpha 0=0$

$\Rightarrow N(A)$ is closed under scalar multiplication

(Q4) [4 *points*] Let $x_1 = (1,1,1)^T, x_2 = (1,2,2)^T, x_3 = (1,2,3)^T, x_4 = (2,2,1)^T, x_5 = (2,3,2)^T$. Find a basis of $\mathbb{R}^3$ from the set $\{x_1, x_2, x_3, x_4, x_5\}$. Justify your answer.

Solution: we need three L.I vectors. $\boxed{4}$

take $\{x_1, x_2, x_3\} \Rightarrow |x_1 \; x_2 \; x_3| = 1 \neq 0 \xrightarrow{L.I}$ it is a basis

OR take $\{x_1, x_2, x_4\} \Rightarrow |x_1 \; x_2 \; x_4| = -1 \neq 0 \xrightarrow{L.I}$ " "

OR take $\{x_1, x_2, x_5\} \Rightarrow |x_1 \; x_2 \; x_5| = -1 \neq 0 \xrightarrow{L.I}$ " "

(Q5 [6 *points*] Let v_1, v_2, v_3 be linearly independent vectors in $\mathbb{R}^4$, and let A be a nonsingular 4×4 matrix. Show that the vectors Av_1, Av_2, Av_3 are linearly independent in $\mathbb{R}^4$

proof: Let $c_1(Av_1) + c_2(Av_2) + c_3(Av_3) = 0$ $\boxed{1}$

then $A(c_1 v_1) + A(c_2 v_2) + A(c_3 v_3) = 0$

$\Rightarrow A(c_1 v_1 + c_2 v_2 + c_3 v_3) = 0$ $\boxed{1}$

but A is nonsingular $\boxed{1}$

$\Rightarrow c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$ $\boxed{1}$

but v_1, v_2, v_3 are L.I. $\boxed{1}$

$\Rightarrow c_1 = c_2 = c_3 = 0$ ✱. $\boxed{1}$