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Birzeit University
Mathematics Department
First Semester 2017/2018
MATH 234 – Quiz 3
Instructor: Dr. Hani Kabajah

Name (بالعربية): ...Key........ **Student No.:** .... **Section:** ....

Question 1 (2 point) Answer by True (T) or False (F)

(1) If $A^2 = I$ then $A^{-1} = A$. (.T.)

(2) The product of two elementary matrices is an elementary matrix. (.F.)

(3) If A is a singular $n \times n$ matrix then the homogenous system $Ax = \mathbf{0}$ has infinitely many solutions. (.T.)

(4) All $n \times n$ matrices have LU-factorization. (.F.)

Question 2 (2 point) Circle the correct answer

(1) The system whose augmented matrix is $\left(\begin{array}{cc|c} 1 & 2 & 1 \\ 3 & a & b \end{array}\right)$ has no solution if

a) $a = 3, b \neq 3$.
b) $a = 6, b \neq 3$.
c) $a = 6, b = 3$.
d) $b = 3$ only.

(2) If $B^T = -B$ then

a) $b_{ii} = 0$ for all i.
b) B is nonsingular.
c) B is singular.
d) B is symmetric.

Question 3 (1 point) Find nonzero 2×2 matrices A and B such that $AB = O$.

$A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

Question 4 (1 point) Find nonzero matrices A, B, and C such that $AC = BC$ and $A \neq B$.

$$AC = BC \Rightarrow AC - BC = 0 \Rightarrow (A-B)C = 0$$

$$A - B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A \neq B \text{ and } AC = BC$$

$$AC = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad BC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Question 5 (2 point) Compute the LU factorization of the matrix $\begin{pmatrix} 3 & 1 \\ 9 & 5 \end{pmatrix}$.

$$A = \begin{pmatrix} 3 & 1 \\ 9 & 5 \end{pmatrix} \xrightarrow[R_2 = -3R_1 + R_2]{} \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix} = U, \quad L = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$$

$$LU = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 9 & 5 \end{pmatrix} = A$$

Question 6 (2 point) If A is nonsingular prove that A^T is nonsingular and $(A^T)^{-1} = (A^{-1})^T$.

A nonsingular $\Rightarrow AA^{-1} = A^{-1}A = I$

Use $(AB)^T = B^T A^T \Rightarrow (AA^{-1})^T = (A^{-1})^T A^T = I$

$$(A^{-1}A)^T = A^T (A^{-1})^T = I$$

$\Rightarrow A^T$ is nonsingular and has the inverse $(A^{-1})^T$

$\Rightarrow (A^T)^{-1} = (A^{-1})^T$