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Birzeit University  
Mathematics Department  
First Semester 2017/2018  
MATH 234 – Quiz 4  
Instructor: Dr. Hani Kabajah

Name (بالعربية): ... *Key* ..... Student No.: ..... Section: .....

Question 1 (6 point) Given the following matrix.

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

- (1) Find  $\det(A)$ .
- (2) Find  $\text{adj } A$ .
- (3) Find  $A^{-1}$ .

(1)  $\det(A) = 3(2 - 2) = 0$

(2)  $\text{adj } A = \begin{pmatrix} 0 & -3 & 3 \\ 0 & 6 & -6 \\ 0 & -3 & 3 \end{pmatrix}^T = \begin{pmatrix} 0 & 0 & 0 \\ -3 & 6 & -3 \\ 3 & -6 & 3 \end{pmatrix}$

(3)  $A^{-1}$  doesn't exist

Question 2 (4 point) Let  $A$  be a nonsingular  $n \times n$  matrix. Show that  $\text{adj } A$  is nonsingular and

$$(\text{adj } A)^{-1} = \det(A^{-1}) A = \text{adj } A^{-1}$$

$$A \text{adj } A = \det(A) I \Rightarrow \det(A) \cdot \det(\text{adj } A) = (\det(A))^{n+1} \stackrel{\det(A) \neq 0}{\Rightarrow} \det(\text{adj } A) \neq 0$$

$\Rightarrow \text{adj } A$  nonsingular

$$\textcircled{1} (A \text{adj } A)^{-1} = (\det(A) I)^{-1}$$

$$\Rightarrow (\text{adj } A)^{-1} A^{-1} = \frac{1}{\det(A)} I \quad / A$$

$$\Rightarrow (\text{adj } A)^{-1} = \frac{1}{\det(A)} A$$

But  $AA^{-1} = I \Rightarrow \det(A) \cdot \det(A^{-1}) = 1 \Rightarrow \det(A^{-1}) = \frac{1}{\det(A)}, \det(A) \neq 0$

$$\Rightarrow \boxed{(\text{adj } A)^{-1} = \det(A^{-1}) \cdot A}$$

$$\textcircled{2} A/(A^{-1})(\text{adj } A^{-1}) = \det(A^{-1}) I$$

$$\boxed{\text{adj } A^{-1} = \det(A^{-1}) A}$$

From  $\textcircled{1}$  and  $\textcircled{2}$

$$(\text{adj } A)^{-1} = \det(A^{-1}) A = \text{adj } A^{-1}$$

$\square$