

Name.....

Number.....

Section

(Q1) [10 points] Use Gauss-Jordan reduction to solve the system below.

$$\begin{aligned} x_1 - x_2 - x_3 - x_4 - x_5 &= 3 \\ 3x_1 - 3x_2 - 3x_3 + 3x_4 - 3x_5 &= 9 \\ -x_1 + x_2 + x_3 + x_4 &= -2 \end{aligned}$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & -1 & -1 & -1 & 3 \\ 3 & -3 & -3 & 3 & -3 & 9 \\ -1 & 1 & 1 & 1 & 0 & -2 \end{array} \right]$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & -1 & -1 & -1 & 3 \\ 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{array} \right] \quad (2)$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & -1 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{array} \right] \quad (2)$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{array} \right]$$

$$\begin{aligned} x_2 &= t & x_5 &= -1 & t, w &\in \mathbb{R} \\ x_3 &= w & x_4 &= 0 & & \\ x_1 &= 2 + t + w & & & & \end{aligned} \quad (5)$$

(Q2) [10 points] Consider the linear system whose augmented matrix is $\left[\begin{array}{ccc|c} 1 & 2 & 0 & a \\ 2 & 5 & b & 1 \\ 1 & 1 & c & 1 \end{array} \right]$

(a) For what values of a, b, c will the system have infinitely many solutions?

(b) For what values of a, b, c will the system have a unique solution?

(c) For what values of a, b, c will the system be inconsistent?

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & a \\ 0 & 1 & b & 1-2a \\ 0 & -1 & c & 1-a \end{array} \right] \quad \textcircled{2}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & a \\ 0 & 1 & b & 1-2a \\ 0 & 0 & b+c & 2-3a \end{array} \right] \quad \textcircled{2}$$

(a) $b+c=0 \ \& \ 2-3a=0$
 $b=-c \ \& \ a=\frac{2}{3}$ \textcircled{2}

(b) $b+c \neq 0 \Rightarrow b \neq -c, \ a \in \mathbb{R}$ \textcircled{2}

(c) $b+c=0 \ \& \ 2-3a \neq 0$
 $b=-c \ \& \ a \neq \frac{2}{3}$ \textcircled{2}
