

Use Gauss-Jordan reduction to solve the following system.

$$\begin{aligned}x_1 + 3x_2 + x_3 + x_4 &= 3 \\2x_1 + -2x_2 + x_3 + 2x_4 &= 8 \\3x_1 + x_2 + 2x_3 + -x_4 &= -1\end{aligned}$$

$$\left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 3 \\ 2 & -2 & 1 & 2 & 8 \\ 3 & 1 & 2 & -1 & -1 \end{array} \right) \rightarrow \left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 3 \\ 0 & -8 & -1 & 0 & 2 \\ 0 & -8 & -1 & -4 & -10 \end{array} \right)$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 3 \\ 0 & -8 & -1 & 0 & 2 \\ 0 & 0 & 0 & -4 & -12 \end{array} \right) \rightarrow \left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 3 \\ 0 & 1 & 1/8 & 0 & -1/4 \\ 0 & 0 & 0 & 1 & 3 \end{array} \right)$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1/8 & 0 & -1/4 \\ 0 & 0 & 0 & 1 & 3 \end{array} \right) \rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 5/8 & 0 & 3/4 \\ 0 & 1 & 1/8 & 0 & -1/4 \\ 0 & 0 & 0 & 1 & 3 \end{array} \right)$$

$$\Rightarrow x_4 = 3, \quad x_3 = \alpha, \quad x_2 = -\frac{1}{4} - \frac{1}{8}\alpha, \quad x_1 = \frac{3}{4} - \frac{5}{8}\alpha$$

$$\Rightarrow \text{The solution of the system} = \left\{ \left(\frac{3}{4} - \frac{5}{8}\alpha, -\frac{1}{4} - \frac{1}{8}\alpha, \alpha, 3 \right) : \alpha \in \mathbb{R} \right\}$$