

Math 234
Quiz 2

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Is it possible to find a 2×2 matrix such that
 A is symmetric, $A^2 = 0$, $A \neq 0$

Solution

Let A be symmetric $\Rightarrow A = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$, $a, b, c \in \mathbb{R}$

$A \neq 0 \Rightarrow a, b, c$ are not all zeros

$$A^2 = 0 \Rightarrow \begin{pmatrix} a & b \\ b & c \end{pmatrix} \begin{pmatrix} a & b \\ b & c \end{pmatrix} = \begin{pmatrix} a^2 + b^2 & ab + bc \\ ab + bc & b^2 + c^2 \end{pmatrix} \\ = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} a^2 + b^2 = 0 \Rightarrow a = 0 \text{ and } b = 0 \\ b^2 + c^2 = 0 \Rightarrow b = 0 \text{ and } c = 0 \end{cases}$$

$\Rightarrow a, b, c$ are all zeros

The answer is No. we can't find $A \neq 0$
such that A is symmetric and $A^2 = 0$