key (Form A)

Berzeit University

Department of Mathematics

Quiz 2

Math 234 October 10, 2018

Name:................. Number:..............

Question One [7 pts]. True or False?

F 1. For every invertible $n \times n$ matrix A there exists a nonzero $n \times n$ matrix B such that AB is the zero matrix.

T 2. The matrix $\begin{bmatrix} 8 & -k \\ k & 1 \end{bmatrix}$ is invertible for all real numbers k.

T 3. Let A be a 3×4 matrix. If $b = a_1 + a_2 + a_3 + a_4$ then the linear system $Ax = b$ has infinitely many solutions.

T 4. If A is an $n \times n$ nonsingular matrix, then A^T is nonsingular.

T 5. If A and B are symmetric $n \times n$ matrices. Then, $AB = BA$ if and only if AB is also symmetric.

F 6. Let A and B be an $n \times n$ matrices. If $AB = O$, then $BA = O$.

T 7. If A is an $n \times n$ matrix such that $A^2 = A$, then $I + A$ is nonsingular and $(I + A)^{-1} = I - \frac{1}{2}A$.

Question Two [3 pts]. Prove the following statements.

(a) If A and B are symmetric $n \times n$ matrices, then ABA must be symmetric.

(b) If two $n \times n$ matrices A and B commute, then A^2 and B must commute as well.

(c) If A is a square matrix satisfying the equation $(A + I)^2 = 0$, then A must be invertible.

Good Luck

Question one

1) $AB = 0 \Rightarrow A^{-1}(AB) = A^{-1}(0)$

$\Rightarrow IB = 0$

$\Rightarrow B = 0$

$\therefore$ B must be zero matrix.

2) $\begin{vmatrix} 8 & -k \\ k & 1 \end{vmatrix} = 8 + k^2 \neq 0, \forall k \in \mathbb{R}$.

$\therefore \begin{bmatrix} 8 & -k \\ k & 1 \end{bmatrix}$ is invertible $\forall k \in \mathbb{R}$.

3) the system $Ax = b$ is consistent since $x = (1,1,1,1)^T$ is a solution. Since A only has 3 rows, then the system can have at most 3 lead variables. therefore, there must be at least one free variable. A consistent system with a free variable has infinitely many solution.

4) $\det(A^T) = \det(A) \neq 0$ (since A is nonsingular)

$\Rightarrow \det(A^T) \neq 0 \Rightarrow A^T$ is nonsingular.

5) ($\Rightarrow$) $(AB)^T = B^T A^T = BA = AB \Rightarrow AB$ is symmetric

($\Leftarrow$) $(AB)^T = AB \Rightarrow B^T A^T = AB \Rightarrow BA = AB$.

(6) $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \Rightarrow AB = 0$ but $BA \neq 0$.

(7) $(I + A)(I - \frac{1}{2}A) = I - \frac{1}{2}A + A - \frac{1}{2}A^2 = I + \cancel{\frac{1}{2}A} - \cancel{\frac{1}{2}A} = I$

$$(I-\tfrac{1}{2}A)(I+A) = I + A - \tfrac{1}{2}A - \tfrac{1}{2}A^2$$
$$= I + \tfrac{1}{2}\cancel{A} - \tfrac{1}{2}\cancel{A} \quad (\text{since } A^2 = A).$$
$$= I$$

$$\Rightarrow (I+A)(I-\tfrac{1}{2}A) = (I-\tfrac{1}{2}A)(I+A) = I$$

$\therefore$ $(I+A)$ is invertible & $(I+A)^{-1} = I - \frac{1}{2}A$.

Question Two

(a) $(ABA)^T = A^T B^T A^T$
$= ABA$ (since A, B symmetric).

(b) Given that $AB = BA$.

Now, $A^2 B = A(AB) = A(BA) = (AB)A$
$= (BA)A = B(AA)$
$= BA^2$.

$$\Rightarrow A^2 B = BA^2$$

(c) $(A+I)^2 = O \Rightarrow A^2 + 2A + I = O$

$\Rightarrow (-A-2I)A = I$ and $A(-A-2I) = I$

$\therefore$ A is invertible and

$$\boxed{A^{-1} = -A-2I}$$

key (Form B)

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Question One [7 pts]. True or False?

F 1. For every invertible $n \times n$ matrix A there exists a nonzero $n \times n$ matrix B such that AB is the zero matrix.

F 2. The matrix $\begin{bmatrix} 8 & k \\ k & 1 \end{bmatrix}$ is invertible for all real numbers k.

F 3. Let A be a 3×4 matrix. If $b = a_1 + a_2 + a_3 + a_4$ then the linear system $Ax = b$ has a unique solution.

F 4. If A is an $n \times n$ nonsingular matrix, then A^T is singular.

T 5. If A and B are symmetric $n \times n$ matrices. Then, $AB = BA$ if and only if AB is also symmetric.

F 6. Let A and B be an $n \times n$ matrices. If $AB = O$, then $BA = 0$.

F 7. If A is an $n \times n$ matrix such that $A^{-1} = A$, then A must be equal to either I or $-I$.

Take $A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$, $A^{-1} = A$ (check) but $A \neq I$, $A \neq -I$.

Question Two [3 pts]. Prove the following statements.

(a) If A and B are symmetric $n \times n$ matrices, then ABA must be symmetric.

(b) If two $n \times n$ matrices A and B commute, then A^2 and B must commute as well.

(c) If A is a square matrix satisfying the equation $(A + I)^2 = 0$, then A must be invertible.

Ans. (a) $(ABA)^T = A^T B^T A^T = ABA$ since A & B are sym.

(b) $A^2 B = A(AB) = A(BA) = (AB)A = (BA)A = B(AA) = BA^2$.

(c) $(A+I)^2 = 0 \Rightarrow A^2 + 2A + I = 0$

$\Rightarrow A(-A-2I) = I$ and $(-A-2I)A = I$

$\Rightarrow A$ is invertible and $A^{-1} = -A-2I$.

Good Luck