Birzeit University

Department of Mathematics

Quiz 4

Math 234 **December 1, 2018**

Name:...Key...... **Number:**..............

Q1 [7.5 points]. Circle the correct answer.

1. If v_1, v_2, v_3 are linearly independent vectors in a vector space V and if $v \in V$ does not belong to $\text{Span}\{v_1, v_2, v_3\}$, then

   (a) $\{v_1, v_2, v_3, v\}$ is linearly dependent

   (b) $\{v_1, v_2, v_3, v\}$ is a spanning set for V

   (c) $\{v_1, v_2, v_3, v\}$ is linearly independent

   (d) dim V ≤ 3

2. The vectors $\{1, \cos 2x, \cos^4 x - \sin^4 x\}$ in $C[0, 2\pi]$ are

   (a) linearly independent

   (b) linearly dependent

   (c) A basis for $C[0, 2\pi]$

   (d) A spanning set for $C[0, 2\pi]$

3. The set $\{(1,1,1)^T, (1,1,\alpha)^T, (1,\alpha,1)^T\}$ is a basis for $\mathbb{R}^3$ if

   (a) $\alpha \neq 1$ and $\alpha \neq -1$

   (b) $\alpha \neq 1$

   (c) $\alpha \neq -1$

   (d) $\alpha \neq 1$ or $\alpha \neq -1$

4. Let $S = \{(a, b, a+b+2c)^T \,|\, a, b, c \in \mathbb{R}\}$. Then a basis for S is

   (a) $\{(1,0,1)^T, (0,1,1)^T\}$

   (b) $\{(1,0,1)^T, (0,1,1)^T, (0,0,1)^T\}$

   (c) $\{(1,1,1)^T\}$

   (d) $\{(1,0,0)^T, (0,1,0)^T\}$

5. One of the following is **true**

   (a) If dim V $= n$, then any n vectors in V span V

   (b) If dim V $= n$, then any n vectors in V are linearly independent

   (c) If dim V $= n$, then any n linearly independent vectors in V form a basis for V

   (d) P_4 is an infinite-dimensional vector space

Question Two (2.5 pts). Let

$$S = \left\{ p(x) \in P_3 : \int_0^1 xp(x)dx = 0 \right\}.$$

Find a **basis** and **dimension** of S.

Let $p(x) \in P_3 \Rightarrow p(x) = ax^2 + bx + c$

Now, $\int_0^1 x\,p(x)\,dx = 0 \Rightarrow \int_0^1 (ax^3 + bx^2 + cx)\,dx = 0$

$\Rightarrow \left. \frac{ax^4}{4} + \frac{bx^3}{3} + \frac{cx^2}{2} \right|_0^1 = 0$

$\Rightarrow \frac{a}{4} + \frac{b}{3} + \frac{c}{2} = 0$

$\Rightarrow \boxed{c = -\frac{1}{2}a - \frac{2}{3}b}$

$\therefore p(x) = ax^2 + bx - \frac{1}{2}a - \frac{2}{3}b$

$= a(x^2 - \frac{1}{2}) + b(x - \frac{2}{3})$

$\therefore S = \text{span}\{x^2 - \frac{1}{2}, x - \frac{2}{3}\}$

$\boxed{\text{a basis of } S \text{ is } \{x^2 - \frac{1}{2}, x - \frac{2}{3}\}}$

and $\boxed{\dim S = 2}$.

Good Luck