Birzeit University
Mathematics Department

Quiz # 3 | Math 234 | 2018/2019

Key

Name.................................. Number................. Section ....

(Q1) [*6 points*] Let A be a 4×4 matrix and assume that $2a_1 - a_3 = a_4$. Determine whether the following statements are true or false.

(a) A is nonsingular. F
(b) A is singular. T
(c) The reduced row echelon form of A is the identity matrix I. F
(d) A cannot be written as a product of elementary matrices. T
(e) The system $Ax = 0$ has infinitely many solutions. T
(f) The system $Ax = b$ is consistent for any $b \in \mathbb{R}^4$ F

(Q2) [*2 points*] Let $A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ -3 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 2 \\ 5 & 0 & -7 \\ -3 & 1 & 5 \end{bmatrix}$

Find an elementary matrix E such that $EA = B$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 - 2R_1$

(Q3) [*4 points*] Find the LU factorization of the matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & 5 \\ -2 & 1 & 0 \end{bmatrix}$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & 5 \\ -2 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & 5 \\ 0 & 4 & 4 \end{bmatrix} \to \begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & 5 \\ 0 & 0 & -16 \end{bmatrix} = U$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 4 & 1 \end{bmatrix}$$

(Q4) [*8 points*] Let $A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$. Find A^{-1}, then use it to solve the system $\begin{matrix} x_1 & + & x_2 & = & -1 \\ 2x_1 & + & 3x_2 & = & -7 \end{matrix}$

$$[A|I] = \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 0 & 3 & -1 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}, \quad X = A^{-1}b = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ -7 \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$$