Birzeit University Mathematics Department

Math 234

2018/2019

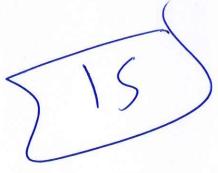
Quiz # 5 + 6 + 7

Name	Number	Section

(Q1) [15 points] Answer by True (T) or False (F)

- (1)... If V is a vector space with dim(V) = 4 and $v_1, v_2, v_3, v_4 \in V$, then span $(v_1, v_2, v_3, v_4) = V$
- (2) ... Let V be a vector space with dim(V) = n and let S be a subspace of V, then 0 < dim(S) < n
- (3) . . Γ . If V is an infinite-dimensional vector space, then any subspace of V is infinite-dimensional.
- (4) ... F. Every linearly independent set of vectors in P_n must contain n polynomials.
- (5) If V is a vector space with dim(V) = n, then any n+1 vectors in V are linearly dependent.
- (6) ... F. If v_1, v_2, \dots, v_n are linearly independent vectors in V, then V is finite dimensional.

- (9) . F. Let $f, g, h \in C^2[a, b]$. If $W[f, g, h](x) = 0 \ \forall \ x \in [a, b]$, then f, g, h are linearly dependent in C[a, b]
- (10) span(1+x, 1-x) is a subspace of P_2
- (11) If $\operatorname{span}(x_1, x_2, x_3) = \mathbb{R}^3$, then $\operatorname{span}(x_1, x_2, x_3, x) = \mathbb{R}^3$ for any $x \in \mathbb{R}^3$
- (12) ... Γ . If S is a subspace of V, then any set of vectors in S that spans S also spans V
- (13) . . \leftarrow . If S is a set of linearly independent vectors in V, then any subset of V containing S is also linearly independent.
- (14) If S is a set of linearly independent vectors in V, then any nonempty subset of S is also linearly independent.
- (14) If S is a subspace of a vector space V, then S is also a vector space.
- (15) If $\{x_1, x_2, x_3\}$ is a subset of a vector space V and $\operatorname{span}(x_1, x_2) = \operatorname{span}(x_1, x_2x_3)$, then x_1, x_2, x_3 are linearly dependent in V.
- (16) The sum of two triangular matrices is triangular.
- (17) If S and T are subspaces of a vector space V, then $S \cap T$ is a subspace of V

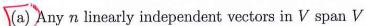


(Q2) [20 points] Circle the correct answer.

(1) One of the following is not a basis of P_3

- (a) $\{1, 2x, x^2 x\}$
- (b) $\{x-1, x^2+1, x^2-1\}$
- (c) $\{x, x^2 + 3, x^2 5\}$
- (d) $\{x^2+1, x^2-1, 2\}$

(2) If V is a vector space with dim(V) = n, then



(b) Any spanning set of V must contain at most n vectors

(c) Any set in V containing less than n vectors must be linearly independent.

(d) All of the above.

(3) $\dim(\text{span}(1-x, x^2, 3+x^2, 1+x^2)) =$

- (a) 0
- (b) 1
- (c) 2
- (d) 3

(4) One of the following sets is linearly independent in P_3

- (a) $\{2, 2-x, x\}$
- (b) $\{2x, 2-x, x^2\}$
- (c) $\{1+x, 1-x, 1\}$
- (d) $\{x, x^2, 2x + 3x^2\}$

(5) Suppose that a vector space V contains n linearly independent vectors, then

(a) Any n vectors in V are linearly independent.

(b) Any set in V containing more than n vectors is linearly dependent.

(c) If S is a set spanning V, then it must contain at least n vectors.

(d) If S is a set spanning V, then it must contain at most n vectors.

(6) Suppose that the set $\{v_1, v_2, v_3\}$ is linearly independent in a vector space V, then

(a) The set $\{v_1, v_1 + v_2, v_2 + v_3\}$ is linearly independent in V

(b) The set $\{v_1 + v_2, v_1 + v_3, v_2 + v_3\}$ is linearly independent in V

(c) The set $\{v_1, v_2, v_1 + v_2 + v_3\}$ is linearly independent in V

(d) All of the above.

(7) One of the following is a subspace of \mathbb{R}^{n}
(a) All triangular $n \times n$ matrices.
(b) All singular $n \times n$ matrices.
(c) All upper triangular $n \times n$ matrices.
(d) All nonsingular $n \times n$ matrices.
(8) One of the following is not a subspace of P_3 (a) $\{p(x) \in P_3 \mid p(2) = 0\}$
(b) $\{p(x) \in P_3 \mid p(1) = p(-1)\}$
(c) $\{p(x) \in P_3 \mid p(0) = 2\}$
(d) $\{p(x) \in P_3 \mid p(2) = p(5)\}$
 (9) Let S = {p(x) ∈ P₃ p(0) = 0 and p(1) = 0}. One of the following is a basis of S (a) {1, x, x²} (b) {x, x²}
(c) $\{x^2 - 1\}$
$\boxed{\text{(d)}} \left\{ x^2 - x \right\}$
(10) The set $\{(1,1,1)^T, (1,1,c)^T, (1,c,1)^T\}$ is a basis of \mathbb{R}^3 if
(a) $c \neq 1$ and $c \neq -1$
$(b)c \neq 1$
(c) $c \neq -1$
(d) $c = 1$ or $c = -1$
(Q3) [5 points] Let A be an $m \times n$ matrix. Show that the null space of A is a subspace of \mathbb{R}^n
problem $N(A) = \begin{cases} x \in \mathbb{R}^n \mid Ax = 0 \end{cases}$ (i) $O \in N(A)$ be cause $AO = O$ \Rightarrow $N(A) \neq \emptyset$ (i) $O \in N(A)$ be cause $AO = O$ \Rightarrow $Ax = O \notin Ay = O$
(i) OFN(A) because Ac (ii) If $x,y \in N(A)$, then $Ax=0$ of $Ay=0$ (ii) If $x,y \in N(A)$, then $Ax=0$ of $Ay=0$ and so $A(x+y) = Ax+Ay = 0+0=0$ $\Rightarrow N(A)$ is closed under $+$ $\Rightarrow N(A)$ is closed under $+$ $\Rightarrow N(A)$ is closed under $+$ $\Rightarrow N(A)$ is closed under $+$
AX=ORAY=01
(ii) IC x, y ENCA), she
(11) +1 · · · · · · · · · · · · · · · · · ·
$\Lambda(\alpha + \gamma) = Ax + \gamma$
and so the
5) par i closed unal
DNA)
Will STALL WIAT & XEIKE Then FIX
(iii) IT XENIA) & XEIR then AX=0 (iii) IT XENIA) & XEIR then AX=0 (iv) = XENIA) & XEIR then AX=0
(1 ii) IT X+NIA) FOXE ME AXZ X 020 So A(XX)=X AXZ X 020 Scalar pultylish

-) NCA) is 3 closed under scalar multiplishes

(Q4) points Let $x_1 = (1, 1, 1)^T$, $x_2 = (1, 2, 2)^T$, $x_3 = (1, 2, 3)^T$, $x_4 = (2, 2, 1)^T$, $x_5 = (2, 3, 2)^T$. Find a basis of \mathbb{R}^3 from the set $\{x_1, x_2, x_3, x_4, x_5\}$. Justify your answer.

Solution we need three L.T vectors.

take $\{x_1, x_1, x_3\} \Rightarrow |x_1 x_2 x_3| = 1 \neq 0 \stackrel{L}{\Rightarrow} i \uparrow i \Rightarrow a \Rightarrow ii)$ The fake $\{x_1, x_1, x_3\} \Rightarrow |x_1 x_2 x_4| = -1 \neq 0 \stackrel{L}{\Rightarrow} i \stackrel{L}{\Rightarrow} i \Rightarrow i \Rightarrow a \Rightarrow ii)$ of fake $\{x_1, x_2, x_3\} \Rightarrow |x_1 x_2 x_3| = -1 \neq 0 \Rightarrow iii$ of fake $\{x_1, x_2, x_3\} \Rightarrow |x_1 x_2 x_3| = -1 \neq 0 \Rightarrow iii$

(Q5 [$\not b$ points] Let v_1, v_2, v_3 be linearly independent vectors in \mathbb{R}^4 , and let A be a nonsingular 4×4 matrix. Show that the vectors Av_1, Av_2, Av_3 are linearly independent in \mathbb{R}^4

prod: Let ((AVI)+(2(AV2)+(3(AV3)=0)

then A((1V1)+A((2V2)+A((3V3)=0)

A((1V1+(2V2+(3V3)=0)

but A1) nensurplar

C(1V1+(2V2+(3V3=0)

but V1, V2, V3 are L. I. I)

but V1, V2, V3 are L. I. I)