100/100

Birzeit University
Department of Mathematics
math 243

Second hour exam — Fall 2013.
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Question #1(35%) Prove or disprove each of the following statements

a)If $A\times B=\phi$ then $A=B=\phi$ (F)

let $A=\{1,2,3\}$
$B=\{\ \}$
$A\times B=\phi$
but $A\neq B$ $A\neq\phi$

b)If R is a relation and $R^{-1}\subseteq R$ then R is symmetric (T)

by Theorem $\Rightarrow$ if $R\subseteq S$ then $R^{-1}\subseteq S^{-1}$

Suppose $R^{-1}\subseteq R$
$\Rightarrow (R^{-1})^{-1}\subseteq R^{-1}$
$\Rightarrow R\subseteq R^{-1}$
$\Rightarrow R=R^{-1}$

R is symmetric iff $R=R^{-1}$

let $(x,y)\in R$
$\Rightarrow (y,x)\in R^{-1}$
$\Rightarrow (y,x)\in R$ (since $R=R^{-1}$).
$\Rightarrow$ R is symmetric.

c)If R is a relation and R is transitive then R^{-1} is transitive (T)

suppose R is a relation and R is transitive and let $(x,y)\in R^{-1}$ and $(y,z)\in R^{-1}$
$\Rightarrow (y,x)\in R$ and $(z,y)\in R$.
$\Rightarrow (z,y)\in R$ and $(y,x)\in R$
$\Rightarrow (z,x)\in R$ (since R is transitive)
$\Rightarrow (x,z)\in R^{-1}$
So R^{-1} is transitive

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5) 1) Is R an equivalence relation

check the three condition:

1) Reflexive: Yes, since $(1,1), (2,2), (3,3), (4,4) \in R$

2) Symmetric: Yes if $aRb \in R$ then $bRa \in R$

since $(1,2), (2,1), (1,3), (3,1), (3,2), (2,3) \in R$

3) Transitive: Yes, if $aRb \in R$ and $bRc \in R$ so $aRc \in R$

since $(1,2), (2,1) \in R$ so $(1,1) \in R$

and $(1,3), (3,1) \in R$ so $(1,1) \in R$

and $(3,2), (2,3) \in R$ so $(3,3) \in R$

and $(2,1), (1,2) \in R$ so $(2,2) \in R$

and $(3,1), (1,3) \in R$ so $(3,3) \in R$

and $(2,3), (3,2) \in R$ so $(2,2) \in R$

since it is Reflexive, Symmetric, Transitive so R is an equivalence relation

2) $R[1] = \{1, 2, 3\}$

3) $R^{-1}\{1, 2\} = \{1, 2, 3\}$

d) If R and S are relations on A then $R \circ S = S \circ R$. (F)

let . counterexample.

$R = \{(1,6), (6,5)\}$

$S = \{(3,1), (5,4)\}$

$RoS = \{(3,6)\}$ $\Rightarrow$ So $RoS \neq SoR$

$SoR = \{(6,4)\}$ $\quad \{(3,6)\} \neq \{(6,4)\}$.

e) If R,S are transitive then RoS is transitive (F)

let $R = \{(1,2), (3,4)\}$ is transitive

$S = \{(6,1), (2,3)\}$ is transitive

$RoS = \{(6,2), (2,4)\}$ is not transitive Because $(6,2) \in RoS \wedge (2,4) \in RoS$ but $(6,4) \notin RoS$

f) If R is transitive the $R \circ R$ is Transitive (T)

Suppose R is transitive and let $(x,y) \in RoR$ and $(y,z) \in RoR$

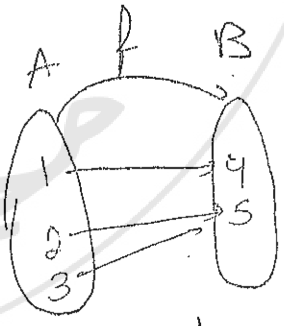
$\Rightarrow \exists s ; (x,s) \in R \wedge (s,y) \in R$ and $\exists t ; (y,t) \in R \wedge (t,z) \in R$

$\Rightarrow (x,y) \in R$ and $(y,z) \in R$ (Since R is transitive).

$\Rightarrow (x,z) \in RoR$ (because $\exists y ; (x,y) \in R \wedge (y,z) \in R$)

So. RoR is transitive

g) If f is a function then f^{-1} is a function (F)



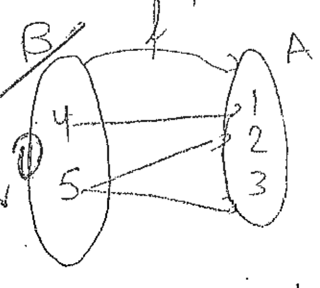
let $f = \{(1,4), (2,5), (3,5)\}$

$f^{-1} = \{(4,1), (5,2), (5,3)\}$

$(5,2) \in f^{-1}$ and $(5,3) \in f^{-1}$

but $2 \neq 3$

So f^{-1} is not function.



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So it is not function

Question#2(15) Which of the following statements is true and which is false

1)...T....If $I_A \subseteq R$ Then R is reflexive on A

2)...F....{ (a,b),(c,d),(a,e) } is afunction

3)...T....*If* A={1,2,3} , Then the number of functions from A to A is 27.

4)...F.... Every relation is a function

5)...T....There is no one to one function from A={1,2,3} onto B={a,b,c,d}

6)...F....If R and S are transitive then R ∪ S is transitive.

7)...F....Any reflexive relation on a set X is transitive

8)...T....There is no onto function from A={1,2,3} onto B={a,b,c,d}

9)...F....If R is reflexive and transitive then R is symmetric.

10)...F.....$(R \circ S)^{-1} = R^{-1} \circ S^{-1}$

Question#3(16%) Let A= ℤ×ℤ* , Let R be a relation on A defined as follows

(a,b) R (c,d) iff ad=bc

a) Show that R is an equivalence relation

$\boxed{1}$ reflexive

let $(a,b) R (a,b)$

$\Rightarrow ab = ba$ ✓ $\forall (a,b) \in A$.

$\boxed{2}$ Symmetric .

$(a,b) R (c,d)$

$\Rightarrow ad = bc$

$\Rightarrow cb = da$

$\Rightarrow (c,d) R (a,b)$.

$\boxed{3}$ transitive .

$(a,b) R (c,d)$ and $(c,d) R (s,t)$

$ad = bc$ and $ct = ds$

$ct = ds$

$\frac{a\cancel{d}}{b} t = \cancel{d}s$??

$at = bs$

$\Rightarrow (a,b) R (s,t)$.

$\Rightarrow$ So R is equivalence relation

b)Find R[(1,2)]

$\{ (c,d) \in ℤ \times ℤ^* \,/\, d = 2c \}$.

15 + 15

Question #4(16%) Use mathematical induction to prove that

$$3^n \geqslant 1+2^n \quad \text{for every } n \in \mathbb{N}$$

1 - The statment is true for n=1

$3^1 \geqslant 1+2$

$3 \geqslant 3$ ✓

2 - suppose the statment is true for n=K

i.e $3^K \geqslant (1+2^K)$

and prove that the statment is true for n=K+1

i.e $3^{K+1} \geqslant (1+2^{K+1})$

Now; L.H.S

$3^{K+1} = 3^K 3 \geqslant (1+2^K)(1+2)$

$= 1+2+2^K+2^K 2$

$= 1+2^{K+1}+2+2^K$

$\geqslant 1+2^{K+1}$ (Since $2+2^K > 0$

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Question #5(18%)

a)Let $f : A \to B$, $g : B \to A$ be functions such that $f \circ g = I_B$. Prove that f maps A onto B

Suppose $f \circ g = I_B$ and let $b \in B$

$\Rightarrow (b,b) \in I_B$

$\Rightarrow (b,b) \in f \circ g$ (since $I_B = f \circ g$).

$\Rightarrow \exists a \in A$; $(b,a) \in g$ and $(a,b) \in f$

$\Rightarrow f(a) = b$

So f is maps A onto B.

b)Let $g : A \to B$, $f : B \to C$ be one to one functions. Show that $f \circ g : A \to C$ is one to one

Suppose $g: A \to B$ and $f: B \to C$ be one to one functions.

and suppose $f \circ g(x) = f \circ g(y)$

$\Rightarrow f(g(x)) = f(g(y)$

$\Rightarrow g(x) = g(y)$ (since f is 1-1)

$\Rightarrow x = y$ (since g is 1-1).

So $f \circ g$ is one to one.

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