

Birzeit University
 Mathematics Department
 Math 234

(97)

First Hour Exam

First semester 2013/2014

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Question #1 (24%): Which of the following statements is true and which is false:

False 1- If A is an $n \times n$ matrix and the system $AX=0$ has a nontrivial solution then A is nonsingular.

True 2- If A and B are $n \times n$ matrices and AB is nonsingular then both A and B are nonsingular.

False 3- If A is 4×4 matrix then $|-A| = -|A|$

False 4- If A is 3×3 matrix and $|A| = -|A^T|$ then A is singular.

False 5- The product of two symmetric $n \times n$ matrices is symmetric.

True 6- If A is a nonsingular matrix then the matrix A^T is nonsingular also.

False 7- Any non homogeneous system of linear equations that has a nontrivial solution must have infinite number of solutions.

False 8- If A, B, C are 2×2 matrices with $AB = AC$ then $B = C$.

False 9- If A and B are 2×2 matrices such that $A \cdot B = 0$ then $A = 0$ or $B = 0$. 50/50

True 10- If $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 0 & 1 \end{pmatrix}$ then $A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & 0 & 1 \end{pmatrix}$

True 11- If A, B are 3×3 matrices with $|A| = 4$, $|B| = 5$ then $|2A^{-1}B| = 10$

True 12- If $A = \begin{pmatrix} 2 & 5 & 7 \\ 1 & 3 & 4 \\ 2 & 1 & 6 \end{pmatrix}$ then the (2, 3) entry of A^{-1} is $\frac{1}{3}$

$$A_{32}^{-1} = \frac{A_{32}}{|A|} = \frac{1}{3}$$

$$\left[\begin{array}{ccc} 2 & 5 & 7 \\ 1 & 3 & 4 \\ 2 & 1 & 6 \end{array} \right] \xrightarrow{R_3 + 2R_2} \left[\begin{array}{ccc} 2 & 5 & 7 \\ 1 & 3 & 4 \\ 4 & 3 & 12 \end{array} \right]$$

$$\left| \begin{array}{ccc} 2 & 5 & 7 \\ 1 & 3 & 4 \\ 4 & 3 & 12 \end{array} \right| = 2 \cdot 1 \cdot 12 - 1 \cdot 1 \cdot 12 = 24 - 12 = 12$$

True 13- If the coefficient matrix of the system $AX=b$ is $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 0 & 4 & 1 \end{pmatrix}$

Then the system must have a unique solution

True 14- Any nonsingular matrix can be written as a product of elementary matrices.

False 15- The product of two elementary matrices is elementary

True 16- $|AB| = |BA|$ for any two $n \times n$ matrices A and B

$$|A| \cdot |B| = |B| \cdot |A|$$

Question #2(30%): Circle the correct answer:

1- If $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$, and $|A| = 6$, and $B = \begin{bmatrix} 2a & 2b & 2c \\ 4g & 4h & 4i \\ -d & -e & -f \end{bmatrix}$ then $|B| =$

a) 48 ~~48~~ c) -24 d) 24

2- If A, B are two $n \times n$ matrices then

a) $\det(AB^T) = \det(AB) \rightarrow |A||B^T| = |A||B|$

b) $\det(\alpha A) = \alpha \det(A)$

c) $\det(A+B) = \det(A) + \det(B)$

3- If $AX=b$ has no solution where A is an $n \times n$ matrix and b is an $n \times 1$ matrix then

a- A is row equivalent to I_n . b- A is nonsingular.

c- A is a product of elementary matrices. d) $AX=0$ has infinitely many solutions,

4- The conditions on a,b such that the system

$$\begin{array}{l} ax + y = 1 \\ 2x + y = b \end{array} \rightarrow \left[\begin{array}{cc|c} a & 1 & 1 \\ 2 & 1 & b \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 2 & 1 & b \\ a & 1 & 1 \end{array} \right]$$

has infinite number of solution is

a) $a=2$ and $b=1$

c) $a=2$ and $b \neq 1$

b) $a \neq 2$ and $b=1$

d) $a \neq 2$ and $b \neq 1$

5- If A and B are $n \times n$ nonsingular matrices then :

a- $(AB)^T = A^T B^T$

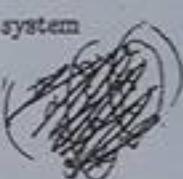
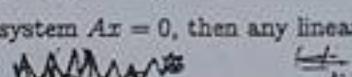
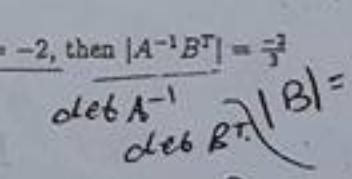
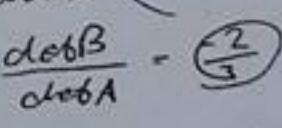
c- $(AB)^{-1} = A^{-1} B^{-1}$

b) $(A+B)^T = A^T + B^T$

d- $|\alpha A| = \alpha^n |A|$

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Question 1 (39 points). Mark each of the following by True or False

- (1) (.T.) An $n \times n$ -matrix A is nonsingular if and only if A is a product of elementary matrices.
- (2) (.F.) Any two $n \times n$ -singular matrices are row equivalent.
- (3) (.T.) If A is a singular matrix and U is the row echelon form of A , then $\det(U) = 0$.
- (4) (.T.) If A is a 3×3 -matrix and the system $Ax = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$ has a unique solution, then the system $Ax = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ has only the zero solution. 
- (5) (.T.) If an $n \times n$ -matrix A is nonsingular, then Cramer's rule can be used to solve the system $Ax = b$.
- (6) (.T.) Let A be a 4×3 -matrix with $a_2 = a_3$. If $b = a_1 + a_2 + a_3$, where a_j is the j th column of A , then the system $Ax = b$ will have infinitely many solutions.
- (7) (.T.) If y, z are solutions to the system $Ax = 0$, then any linear combination of y, z is also a solution to $Ax = 0$. 
- (8) (.T.) If A is a singular and B a nonsingular $n \times n$ -matrices, then AB is singular.
- (9) (.F.) If a matrix B is obtained from A by multiplying a row of A by a real number c , then $|A| = c|B|$.
- (10) (.T.) If A, B are $n \times n$ -matrices, $|A| = 3$ and $|B| = -2$, then $|A^{-1}B^T| = \frac{-2}{3}$.
- (11) (.T.) If A is a singular matrix, then $\text{adj}(A) = 0$. 
- (12) (.T.) $S = \left\{ \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} : a, b \in \mathbb{R} \right\}$ is a subspace of \mathbb{R}^3 . 
- (13) (.F.) If A is row equivalent to B , then $\det(A) = \det(B)$.
- (14) (.F.) If A is a nonsingular $n \times n$ -matrix and c is a nonzero real number, then $(cA)^{-1} = \left(\frac{1}{c}\right)^n A^{-1}$.
- (15) (.F.) If A is a singular $n \times n$ -matrix, $b \in \mathbb{R}^n$, then the system $Ax = b$ has infinitely many solutions.
- (16) (.F.) If A, B are 4×4 -matrices and AB is the zero matrix, then $\det(A) = 0$.

~~AB = 0~~

$$\det(AB) = \frac{1}{\det(A)} \det(B)$$

33